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This report, in the form of a teacher's guide, presents materials for a ninth grade introductory course on Introduction to Quantitative Science (IQS). It is intended to replace a traditional ninth grade general science with a process oriented course that will (1) unify the sciences, and (2) provide a quantitative preparation for the new science materials. The course materials consist of descriptions of about 80 laboratory and demonstration activities which provide the basis for class discussion and approximately 30 instructor information sheets providing directions and suggestions for guiding activities. There are a number of detailed activities for students, a teacher's outline of principles, and problem sheets to be used to direct students beyond the immediate experimental conclusions. Areas dealt with are (1) measurement and fundamental quantities, (2) graphical analysis, (3) properties of light, (4) matter, and (5) rectilinear kinematics. Because the program is a departure from traditional ninth grade general science, and even though the materials are quite complete, it is strongly recommended that teacher inservice education be provided prior to adoption of the program. (DH)

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REPORT
Project No. 6-1708 -24
Contract No. OEC-3-7-061708-2206

INTRODUCTION TO QUANTITATIVE SCIENCE

February 1968

U.S. DEPARTMENT OF
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FINAL REPORT
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INTRODUCTION TO QUANTITATIVE SCIENCE

A Ninth-grade Laboratory-centered Course Stressing Quantitative Observation and Mathematical Analysis of Experimental Results

Lawrence J. Badar

Rocky River Public Schools
Rocky River, Ohio

February 1968

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SUMMARY

In fulfillment of the stated project goal, materials have been developed for a ninth-grade science course which is quantitative, laboratory based, and which emphasizes graphical analysis of experimental results. INTRODUCTION TO QUANTITATIVE SCIENCE (IQS) is proposed to constitute the first course in a structured four-year science program with CHEMS Chemistry, BSCS Biology, and PSSC Physics following sequentially.

IQS is intended to take full advantage of what is common in the objectives, approach, emphasis, and even content of these program curricula while serving as a specific preparation for them.

The investigative nature of the IQS course precludes a student subject-matter textbook in the traditional sense; hence the course syllabus is presented as a teacher guide which suggests some eighty laboratory activities which provide the foundation for the topics treated and extended in discussion. Because of this laboratory orientation, then, the brief subject-matter outline, rather than prescribe a myriad of factual material to be conveyed to the students, _evolves_ from experimental observations.

Areas of investigation judged appropriate to meet the established criteria for IQS are: Measurement and Fundamental Quantities, Graphical Analysis, Properties of Light, Matter, and Rectilinear Kinematics.* While this subject matter, with some topical rearrangement, constitutes the seven units of the IQS course, it must be emphasized that _content_ is considered secondary in importance to the value of the particular investigation and development as an experience in the _processes_ of science.

An integral part of the course development is provided by the set of more than thirty problem sheets, most of which relate directly to laboratory results and are designed to assist the student in progressing logically beyond experimental conclusions.

Since IQS represents a severe departure from courses traditionally offered at the ninth grade level, a dramatic change in the approach, attitude, and function of the instructor is demanded. Despite the intended completeness of the 200-plus page Teacher Guide, the nature of the IQS course prompts the strong recommendation and encouragement that an institute or similar program be made available to prospective IQS teachers.

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*Since the materials for the unit on Rectlinear Kinematics were developed and tested prior to the initiation of this project, they are not included in this report. The unit, however, is recommended as a part of the IQS course.

INTRODUCTION

While BSCS, CBA, CHEMS, and PSSC have initiated substantial curriculum revision in the specific science subject areas, a considerably lesser effort has been directed toward the total secondary school science program. If a smooth transition -- or even strict consistency -- was intended by the originators of the separate programs, that this was achieved is not readily apparent. Also, for schools adopting one or more of these more demanding courses, it is not unreasonable to question the value of traditional General Science as a preparation for a curriculum which represents so severe a departure from tradition.

Considerations of this kind and the obvious difficulty of meaningfully presenting these upgraded investigatory courses within the time limitations imposed by the school year led to both the design of a course at the ninth-grade level and the adoption of a structured sequence for all students anticipating enrollment in four years of high school science.

The special course, called INTRODUCTION TO QUANTITATIVE SCIENCE (IQS), represents a conscious effort to take full advantage of the overlap in approach, emphasis, object and even subject matter in the new biology, chemistry, and physics courses while serving as a specific preparation for them. As the course title implies, measurement serves as a natural unifying theme allowing the introduction of quantitative aspects of chosen subject matter from each of the principal science disciplines.

Basically, IQS differs from General Science in that relatively few topics are covered, but each is treated to a considerable depth and thoroughness. Also, the course is grounded in the laboratory where the rule of approach is "guided discovery" in which the experimental exercises are presented as a problem to be solved rather than a recipe to be followed. Instructions for procedures are minimal and left in large part to the ingenuity of the student. The emphasis is on developing quantitative reasoning rather than mere descriptive fact gathering. The experiments are selected and designed to foster a mode of intelligent inquiry in the student; and ideally, all observations are expressed in some mathematical formulation such as an equation or graph. To this end the subject matter has been so chosen as to permit a laboratory investigation by the student culminating in a mathematical expression of the experimental result, and followed by discussion, extension, and application of the principles discovered.

With full realization that the materials are intended for use at the ninth-grade level, the selection of course content is governed primarily by five general considerations:

(1) The topic must lend itself to initial investigation in the laboratory.

(2) The topic must admit of a graphical, geometrical, or analytical procedure within the grasp of students at this mathematical level.

(3) The topic must relate to a unified whole and not be an entity isolated from a central theme; it must not stop with the laboratory but permit development into a broader concept.

(4) The topic must represent authentic, fundamental science.

(5) The topic must have relevance in the context of later chemistry, physics, or biology courses.

With IQS providing a wealth of experience in laboratory procedures and techniques, the analysis of data and the interpretation and extension of observed results, the teachers of chemistry, biology, and physics can realize a desirable latitude of time, subject matter, and level of presentation in their courses. For example, some of the Advanced Topics materials can be incorporated in the physics course; Laboratory Blocks and Invitations to Enquiry can be added to the biology program. While these possibilities certainly exist, it should be emphasized that it was predominantly the demands of the new curricula *per se* in time and subject matter that created the prime impetus for the development of IQS as a preparatory course.

METHODS

After an initial decision on content areas to be included, course materials were developed and classroom tested through much of the 1966-7 and 1967-8 school years. The pilot group consisted of seventy ninth-grade students who elected INTRODUCTION TO QUANTITATIVE SCIENCE and whose ability and performance levels in mathematics and science were generally above the local average. Because of the preliminary nature of the work, only a subjective evaluation of the effectiveness of the materials was possible.

RESULTS AND FINDINGS

The essence of this curricular effort is embodied in the appended comprehensive Teacher's Guide for the course, INTRODUCTION TO QUANTITATIVE SCIENCE. The laboratory exercises and apparatus, problem sheets, tests, outline, and other materials comprising the six units (an additional unit on Rectilinear Kinematics developed prior to the initiation of this project, is recommended to complete the IQS course) of the quantitative, laboratory-oriented program have been judged to be of merit in establishing a sound foundation for enhanced student performance in CHEMS Chemistry, BSCS Biology, and PSSC Physics courses to follow.

CONCLUSIONS AND RECOMMENDATIONS

While a serious attempt has been made to provide sufficient detail for a teacher to initiate the course, the nature and spirit of IQS materials and course objectives prompt a strong recommendation for a program of teacher orientation. The engagement of additional pilot schools is suggested for a more intense effort of evaluating the materials. The preparation of an effective instrument for validly testing the impact of IQS on total student achievement is also urged.

APPENDIX

TEACHER'S COURSE GUIDE

INTRODUCTION
TO
QUANTITATIVE SCIENCE

PREFACE

to

IQS TEACHER'S GUIDE

The various components of the Guide have been color-coded for ease of access: BLUE for Instructor Information Sheets, YELLOW for Student Laboratory Sheets, PINK for Test and Quiz Sheets, and WHITE for Content Outline Sheets.

In the right margin of the Outline, reference to accompanying laboratory demonstrations and student experiments are noted. A single asterisk indicates an explanatory Instructor Information Sheet; two asterisks designate an accompanying Student Laboratory Sheet.

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THE FIRST DAY

Since the underlying theme of the course is measurement, it was thought that engaging the students in the act of measuring immediately is quite appropriate and even important in creating an impression and establishing an attitude.

One way in which this can be done that has proved both enlightening and interesting is as follows:

(1) at the beginning of the very first class, after the thirty or less seconds required to introduce the teacher and identify the course by title, fever thermometers are distributed to each of the students.

(2) A few simple instructions on procedure previously written on the board or on an overhead transparency will help expedite this activity; these relate to alcohol swabbing, shaking down the thermometer, reading the thermometer, scale graduations, etc.

(3) Within five minutes each exuberant adolescent is quietly heeding the gentle directive to keep the mouth shut for a period of three minutes with the thermometer placed firmly under the tongue; this is an excellent time to call roll --- it has provided a million garbled laughs!

(4) The temperatures are recorded by each student as well as on a prepared sheet which immediately displays the distribution of temperatures for the entire class.

(5) Many models of thermometers supply a standardization chart indicating corrections to the temperature reading, if necessary; this usually requires recording the thermometer serial number and seems to lend "scientific importance" or some such thing to the activity!

(6) A follow-up the next day with a set of pertinent questions subtly but effectively introduces the concepts of mean, median, sample size, "normal temperature", etc.

QUANTITATIVE SCIENCE DATE________________ NAME________________________

DIRECTIONS: Place the appropriate symbol ♀ (female) or ♂ (male) in the first unfilled square by your measured temperature. If your temperature is less than 97.0 °F or greater than 101.0 °F, give the actual temperature instead of the symbol in the proper square.

Less than	97.0 °F														
	97.0 °F														
	97.2 °F														
	97.4 °F														
	97.6 °F														
	97.8 °F														
	98.0 °F														
	98.2 °F														
	98.4 °F														
	98.6 °F														
	98.8 °F														
	99.0 °F														
	99.2 °F														
	99.4 °F														
	99.6 °F														
	99.8 °F														
	100.0 °F														
	100.2 °F														
	100.4 °F														
	100.6 °F														
	100.8 °F														

101.0 °F

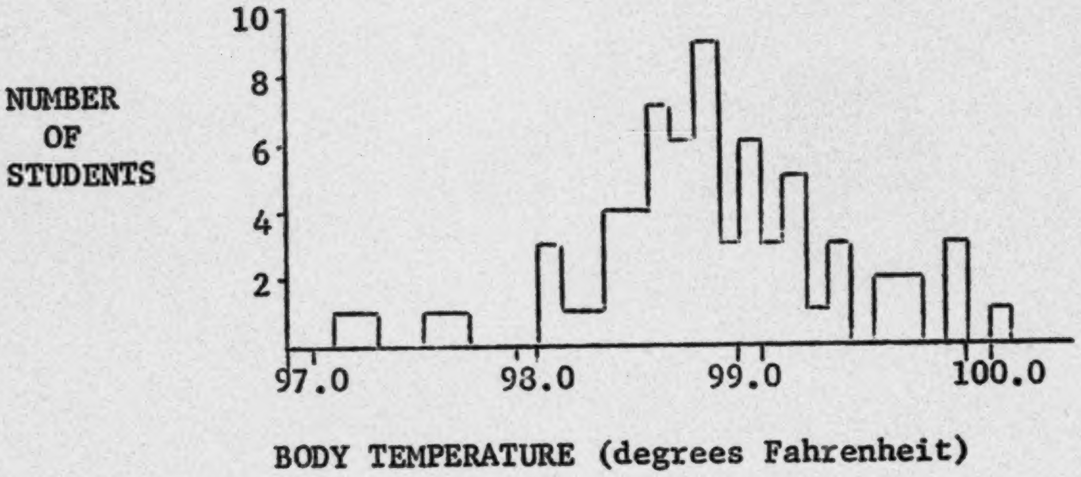
Greater than 101.0 °F

QUANTITATIVE SCIENCE DATE________________ NAME________________________

DISTRIBUTION OF ORAL BODY TEMPERATURES

for Students in Quantitative Science

September 1967



1. Outline the temperature data for students in your class on the bar graph.
2. Find the average temperature for students in your class.
3. How does this compare to the average for all the Quantitative Science students?
4. Compare the median temperature of the students in your class to the median of the whole group.
5. Comment on the "normal" temperature of 98.6°F.

UNIT I. INTRODUCTION

A. Course title: INTRODUCTION TO QUANTITATIVE SCIENCE

1. INTRODUCTION

    a. The course is intended to be fundamental, basic, and an introduction to the concerns, methods, modes, and moods of the scientist.

    b. It follows from the fundamental character of the course that no previous experience in physics, chemistry or biology is required or expected.

    c. Topics are so chosen that they can be introduced by simple, direct, experimental observations.

    d. The background of eight-grade with concurrent ninth-grade mathematics is sufficient; any additional math required will be developed as needed.

    e. Hopefully, the course will make later courses in chemistry, biology, and physics more meaningful.

    f. Hopefully, too, the course will instill a greater awareness and appreciation of science around us; make us scientifically literate citizens of the universe.

2. QUANTITATIVE

    a. Laboratory observations, results, conclusions, and discussions and developments are not merely descriptive, but based on measurement and hence quantitative.

    b. It is within the objective of the course that the idea of measurement become an important part of your thinking.

    c. Observations are expressed mathematically by DATA TABLES, GRAPHS, and EQUATIONS.

    d. Topics are chosen that permit mathematical analysis and interpretation of results at the ninth-grade level of mathematics.

    e. The course is not a survey course in which a large number of topics are treated lightly; instead, the number of topics selected is sharply limited and each is treated to a considerable depth.

3. SCIENCE

    a. Topics are selected from the fields of physics, chemistry, and biology (to a lesser extent).

    b. This is not a course in any one of these, but it is an attempt to introduce principles, methods, attitudes, and reasoning fundamental to all true science.

    c. What is science? Discussion, references, assignment.

UNIT I. INTRODUCTION (continued)

B. Experimental nature of INTRODUCTION TO QUANTITATIVE SCIENCE

1. The subject matter is based entirely on experiments; OBSERVATION IN THE LABORATORY SERVES AS THE BASIS FOR ALL DISCUSSION AND DEVELOPMENT.
2. With few exceptions, nothing will be discussed in this course that has not first been observed in the laboratory or deduced from an experimental result.
3. The course is an attempt to gain experience in thoughtful searching and observing, organizing, measuring, recording, interpreting, analyzing, concluding, and predicting.
4. The course provides an opportunity to develop laboratory skills and techniques.
    - a. Accuracy and care in measurement.
    - b. Appreciation for the instruments of science by understanding and using them.

C. Measurement

1. Definition: COMPARISON WITH A CHOSEN STANDARD
    - a. Measurement is an expression of how many times a given quantity is greater or less than a chosen standard.
    - b. Standards provide the language of measurement, the means by which quantitative information is communicated.
2. IDEAL standards are
    - a. accessible (early emphasis)
    - b. invariable (same every place and at any time)
    - c. exact (capable of extremely accurate comparison)
3. Standards are arbitrarily chosen (example of the GREEN GRUNT as an arbitrary choice of length measure)
4. Multiples and fractions of standards: METRIC PREFIXES

Mega --- one million
Kilo --- one thousand
centi --- one hundredth
milli --- one thousandth
micro --- one millionth

5. The three basic quantities measured are MASS, LENGTH, and TIME.
    - a. MASS, LENGTH, and TIME are fundamental quantities; every event, phenomenon, occurrence, that takes place requires only the elements of MASS, LENGTH, and TIME in its most basic description.
    - b. All other physical quantities (speed, density, force, etc.) can be reduced to a description in terms of MASS, LENGTH, and TIME, and are referred to as derived quantities.

Experiment: HEIGHT DISTRIBUTION

Another activity which requires no previous preparation and hence is well suited for the first few days of the course, is the measurement of students' heights. While this is similar in essence and objective to the first B.S.C.S. Blue Version investigation, several additional features can extend the breadth and interest of the activity.

(1) Recording the measured height on the prepared sheet (or a transparency similarly constructed) immediately displays the height distribution for the entire class.

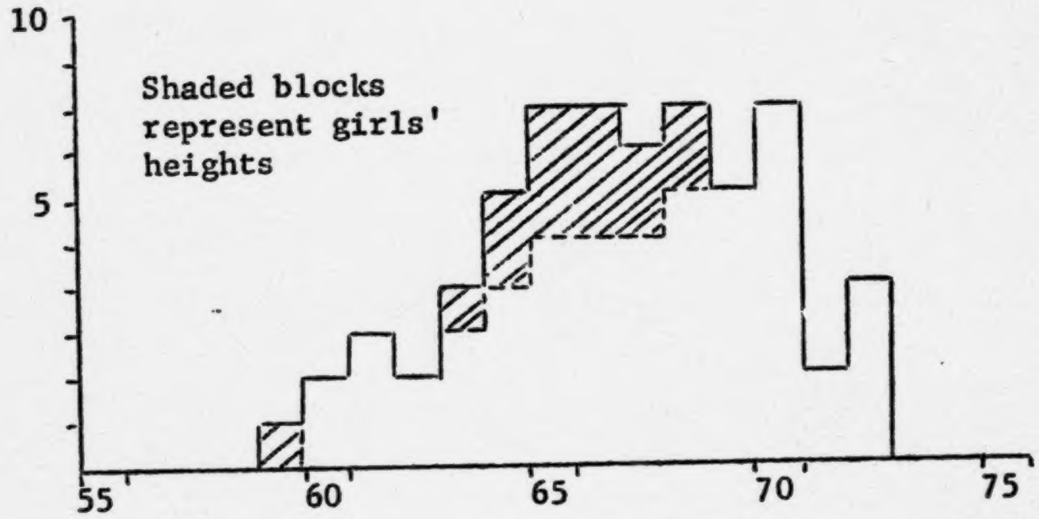
(2) Beprepared for the suggestion to have separate distributions for boys and girls; also separation by ages.

(3) To increase the sample size, combine distributions from several classes (see sample below for some sixty students).

(4) Suggest an even greater sample, the entire (or most) of the ninth-grade class; this is accomplished with little difficulty if meter sticks are mounted vertically on the corridor wall outside the classroom where the Quantitative Science students can conveniently and quickly measure the heights of their assigned subjects.

(5) At the end of the year, each student height is again measured and the new distribution is discussed; repeating this in the senior year for the same students furnishes at least a good laugh!

Height Distribution for Students in Quantitative Science September 1967



QUANTITATIVE SCIENCE DATE__________________ NAME__________________________

DIRECTIONS: Place the appropriate symbol ♀ (female) or ♂ (male) in the first unfilled square by your measured height. If your height is less than 54" or greater than 72", give the actual height instead of the symbol in the proper square.

Less than 54 inches															
54 inches (4'6")															
55 "															
56 "															
57 "															
58 "															
59 "															
60 " (5'0")															
61 "															
62 "															
63 "															
64 "															
65 "															
66 " (5'6")															
67 "															
68 "															
69 "															
70 "															
71 "															
72 " (6'0")															
Greater than 72 inches															

THE SLIDE RULE: SIGNIFICANT FIGURES

Beginning with the instructions on slide rule use, the rest of the course presumes each student has a slide rule with him at all times. Inexpensive plastic or wood rules are available from various local suppliers. Keep in mind that the instructor's job is somewhat simplified if all the students have the same model. Both large demonstration rules and transparent models for overhead projection have been found very effective also.

There are programmed instruction materials available on the use of the slide rule. While these may offer the desirable advantage of saving class time, it is felt that for most students at this age, teacher instruction is a distinct help in instilling the necessary confidence in the operation of the slide rule.

The problem sets and problems within a set are at an attempt at graduated difficulty; all are suitable for both in-class and home exercises.

It should be pointed out that very little is done here on significant figures by formal group instruction. After a brief introduction (see outline notes), a few examples are given and significant figures are treated only casually, usually individually, and almost exclusively referring to calculations based on laboratory measurements. The same thing can be said of ERROR and ACCURACY in general. The reason is simply a belief that formal treatment of these topics at this level carries an assurance of boredom and unusual reluctance to learn!

UNIT I. INTRODUCTION (continued)

D. The Slide Rule

1. Instrument designed for ease and speed in computation.
    - a. Operation of slide rule based on logarithms and properties of logarithms.
    - b. Computational aid for a variety of common (and some uncommon) arithmetic operations.
    - c. Even though the slide rule deals with numbers only to a known approximation, it in no way suggests sloppy operation or handling!

2. Parts
    - a. BODY }
    - b. SLIDE } SCALES on both; usually identified by letter
    - c. CURSOR, or INDICATOR, with hairline

3. The C and D scales are most used and the most useful.
    - a. C and D scales are principal scales for multiplication and division.
    - b. C and D scales are identical in structure, and adjacent in position.
    - c. C scale is on the SLIDE, and hence movable relative to D scale, which is on the BODY.
    - d. LEFT INDEX of each is 1; RIGHT INDEX is also 1 on both C and D scales.
    - e. PRIMARY GRADUATIONS: ten major subdivisions on both C and D scales, consecutively labeled 1, 2, 3, 4, 5, 6, 7, 8, 9, 1.
    - f. C and D scales are nonlinear, that is, the interval between graduations is successively smaller from left to right: (1 - 2) > (2 - 3) > (3 - 4) > (4 - 5) ...
    - g. In reading scales, regard primary graduations as exact whole numbers from 1 through 9.
    - h. Smallest marked division (smd) in each interval:

interval	(LEFT INDEX - 2)	(2 - 4)	(4 - RIGHT INDEX)
smd	.01	.02	.05

    - i. If PRIMARY GRADUATIONS are regarded as giving the units place, then SECONDARY GRADUATIONS give the tenths place, and TERTIARY GRADUATIONS give the hundredths place.
    - j. Read numbers on the slide rule to the hundredths place or three digits.

D. The Slide Rule (continued)

4. Significant figures

a. Useful estimate of the accuracy to which a number is known, expressed by retaining the proper number of digits.

b. Two-digit significance implies that the number is known no better than one part in 100, and no worse than one part in 10.

c. Three-digit significance implies that the number if known no better than one part in 1000, and no worse than one part in 100.

d. Least reliable and doubtful figures; examples in adding or multiplying.

e. Zeroes: before and after.

f. The slide rule preserves three-digit significance.

5. To MULTIPLY a number P by a number Q: 2.00 X 3.00

a. Set the index of the C scale at P on the D scale.

b. Move the indicator hairline to Q on the C scale.

c. Read the product at the indicator on the D scale.

Examples:
4.00 X 2.00
2.50 X 3.50 = 8.75
3.14 X 2.12 = 6.66
1.56 X 4.93 = 7.68
5.07 X 1.75 = 8.87

3.00 X 4.00
1.67 X 8.45 = 14.1
5.62 X 8.45 = 27.7
3.07 X 9.77 = 30.0
5.07 X 1.75 X 9.63 = 85.4

6. To DIVIDE a number P by a number Q: 8.00/4.00

(a. Set the indicator hairline at P on the D scale)

b. Set Q on the C scale at P on the D scale.

c. Move the indicator to the index of the C scale.

d. Read the quotient at the indicator on the D scale.

Examples:
6.80/4.20 = 1.70
9.74/3.71 = 2.63
2.95/1.53 = 1.93
5.96/7.69 = 0.776

THE SLIDE RULE

To MULTIPLY a number P by a number Q: P X Q

1. Set the index of the C scale at P on the D scale
2. Move the indicator to Q on the C scale
3. Read the product at the indicator hairline on the D scale

To DIVIDE a number P by a number Q: P/Q

(1. Set the indicator at P on the D scale)
2. Set Q on the Cscale at P on the D scale
3. Move the indicator hairline to the index of the C scale
4. Read the quotient at the indicator on the D scale

DECIMAL POINT DETERMINATION: Scientific Notation

1. Write all numbers with one digit to the left of the decimal point and the appropriate power of ten
    a. If the decimal point is moved to the left, the exponent of ten is positive and is equal to the number of places moved
    b. If the decimal point is moved to the right, the exponent of ten is negative and is equal to the number of places moved

2. To multiply powers of ten, add the exponents

3. To divide powers of ten, subtract the exponent of the divisor from the exponent of the dividend (subtract the exponent of the denominator from the exponent of the numerator)

   (APPLICATION OF 2 and 3 in MULTIPLE OPERATIONS:
   add the exponents of ten in the numerator
   add the exponents of ten in the denominator
   subtract the sum of the exponents in the denominator from the sum of the exponents in the numerator)

4. Obtain the slide rule result of the indicated operations on the numbers

5. ESTIMATE the result of performing the indicated operations on the numbers. This tells you where to place the decimal point in the result of number 4

6. If necessary, rewrite the answer with one digit to the left of the decimal point (by using 1 and 2 above)

QUANTITATIVE SCIENCE DATE______________ NAME__________________

PERFORM THE INDICATED OPERATIONS ON THE SLIDE RULE; COMPARE WITH THE RESULT OBTAINED BY ORDINARY MULTIPLICATION.

1. 2.70 X 3.00 __________ __________
2. 4.35 X 1.77 __________ __________
3. 3.44 X 1.89 __________ __________
4. 1.65 X 5.85 __________ __________
5. 8.60 X 1.14 __________ __________

6. 1.08 X 6.45 __________ __________
7. 2.06 X 3.24 __________ __________
8. 4.37 X 2.17 __________ __________
9. 3.39 X 3.13 __________ __________
10. 2.28 X 6.10 __________ __________

11. 5.55 X 3.32 __________ __________
12. 6.75 X 4.55 __________ __________
13. 9.57 X 2.31 __________ __________
14. 2.33 X 3.72 __________ __________
15. 6.04 X 1.46 __________ __________

QUANTITATIVE SCIENCE

NAME ____________________

DATE ____________________

PERFORM THE INDICATED OPERATIONS ON THE SLIDE RULE; COMPARE WITH THE RESULT OBTAINED BY LONG DIVISION OR MULTIPLICATION

1. 1.46 X 3.08 X 2.18 ________ ________

2. 2.17 X 4.25 X 1.84 ________ ________

3. 8.45 X 3.58 X 3.30 ________ ________

4. 4.15 X 2.39 X 4.25 ________ ________

5. 3.82 X 1.64 X 8.32 X 1.77 ________ ________

6. 7.20 ÷ 3.16 ________ ________

7. 3.66 ÷ 1.98 ________ ________

8. 8.35 ÷ 7.60 ________ ________

9. 1.96 ÷ 2.84 ________ ________

10. 3.71 ÷ 2.93 ________ ________

11. 2.33 ÷ 3.81 ________ ________

12. 1.64/7.14 ________ ________

13. 9.43/9.58 ________ ________

14. $\frac{7.45 \text{ X } 3.55}{2.23}$ ________ ________

15. $\frac{2.99 \text{ X } 2.61}{9.46}$ ________ ________

D. The slide rule (continued)

7. Multiple operations

a. Extended product: $P \times Q \times R \times \ldots$

(1) after each multiplication, indicator gives partial product on D scale.

(2) leaving indicator in place, move index of C scale to indicator position.

(3) move indicator to next factor on C scale; cumulative product is read at the indicator on the D scale.

Example: $2.22 \times 1.78 \times 2.44 = 9.65$

b. Mixed products and quotients: $\dfrac{P \times Q \times R \times \ldots}{S \times T \times \ldots}$

(1) alternately divide and multiply.

(2) indicator or index gives intermediate result after each operation (multiplication and division, respectively).

(3) read only final answer.

Examples:

$$\frac{9.75 \times 2.89 \times 4.57}{3.22 \times 7.63} = 5.25$$

$$\frac{5.18 \times 1.35 \times 9.28}{4.31 \times 6.47} = 2.32$$

$$\frac{3.82 \times 2.46 \times 4.87}{2.96 \times 1.03} = 14.90$$

QUANTITATIVE SCIENCE DATE______________ NAME________________________

PERFORM THE INDICATED OPERATIONS ON THE SLIDE RULE.

1. 1.54 X 3.46 1____________

2. 2.65 X 3.54 2____________

3. 5.30 X 1.89 3____________

4. 2.33 X 7.97 4____________

5. 8.88 X 6.66 5____________

6. 1.93 X 4.25 X 1.21 6____________

7. 9.45 X 5.76 X 2.83 7____________

8. 3.82 X 2.08 X 3.32 8____________

9. 8.55 X 6.45 X 2.09 9____________

10. 4.75 ÷ 3.64 10____________

11. 71.4 ÷ 44.3 11____________

12. 8.55 ÷ 2.07 12____________

13. 8.02 ÷ 9.35 13____________

14. 714 ÷ 953 14____________

15. $\frac{6.66 \text{ X } 5.86}{4.06}$ 15____________

16. $\frac{5.18 \text{ X } 1.35}{4.31}$ 16____________

17. $\frac{1.07 \text{ X } 1.01}{1.09}$ 17____________

18. $\frac{9.28 \text{ X } 1.35 \text{ X } 5.18}{6.47 \text{ X } 4.31}$ 18____________

19. $\frac{2.91 \text{ X } 6.85 \text{ X } 4.77}{2.32 \text{ X } 2.73}$ 19____________

20. $\frac{3.93 \text{ X } 1.44 \text{ X } 9.55}{5.95 \text{ X } 8.85}$ 20____________

BIG BONUS! $\frac{7.96 \text{ X } 6.97 \text{ X } 2.21 \text{ X } 4.44}{9.42 \text{ X } 5.26 \text{ X } 8.11}$ BB____________

UNIT I. INTRODUCTION (continued)

E. SCIENTIFIC NOTATION or Powers-of-ten Notation

1. A compact method of expressing and working with numbers which relies on the properties of exponents.
    a. Of particular help in handling very large numbers (for example, the 600,000,000,000,000,000,000,000 particles in one cubic foot of air), and very small ones (for example, the 0.000000000000000000000000000002-lb weight of a sub-atomic particle)
    b. Offers a consistent and easy means of using and retaining SIGNIFICANT FIGURES.
    c. Certainly one of the better ways of dealing with the decimal point in slide rule calculations.

q 2. Powers of ten
    a. Positive exponents.

$10 = 10^1$
$100 = 10^1 \times 10^1 = 10^2$; ten used as a factor twice
$1000 = 10^1 \times 10^1 \times 10^1 = 10^3$; ten used as a factor three times.
$10,000 = 10^1 \times 10^1 \times 10^1 \times 10^1 = 10^4$; ten used as a factor four times.
$100,000 = 10^1 \times 10,000 = 10^1 \times 10^4 = 10^5$.
$1,000,000 = 10^2 \times 10^2 \times 10^2 = 10^6$.

    b. Negative exponents.

$0.1 = 1/10 = 1/10^1 = 10^{-1}$
$0.01 = 1/100 = 1/10^2 = 10^{-2}$
$0.000001 = 1/1,000,000 = 1/10^6 = 10^{-6}$

    c. Zero exponent.

$10^0 = 1$; ANY NUMBER TO THE ZERO POWER IS ONE.

3. In Scientific Notation, a number is written with one digit to the left of the decimal point and the appropriate power of ten.
    a. Written in symbols, Scientific Notation can be expressed as $M \times 10^n$, where $1.00 \leq M < 10.0$, and n is an integer, positive, negative, or zero.
    b. EXAMPLES: $212 = 2.12 \times 100 = 2.12 \times 10^2$
    $0.005 = 5/1000 = 5/10^3 = 5. \times 10^{-3}$
    c. Remember, Scientific Notation in no way changes the VALUE of a number; it is an equivalent way of expressing the SAME number.
    d. Incidentally, there are 6×10^{23} particles in one cubit foot of air, and the weight of that sub-atomic particle is 2×10^{-30} lbs!

E. SCIENTIFIC NOTATION (continued)

4. Rules for expressing numbers in Scientific Notation (these are based on 2a, 2b, and 2c above)

   a. If the decimal point is moved to the left, the exponent in the power of ten is positive and is equal to the number of places moved.

   EXAMPLES: $147 = 1.47 \times 10^2$; the decimal point was moved two places to the left, from the right side of the 7 to the right side of the 1.

   $5280 = 5.28 \times 10^3$

   b. If the decimal point is moved to the right, the exponent in the power of ten is negative and is equal to the number of places moved.

   EXAMPLES: $0.0643 = 6.43 \times 10^{-2}$
   $0.0000891 = 8.91 \times 10^{-5}$

   c. To multiply powers of ten, algebraically add the exponents.

   EXAMPLES: $10^1 \times 10^3 \times 10^5 = 10^{1+3+5} = 10^9$
   $10^2 \times 10^{-3} \times 10^4 = 10^{2-3+4} = 10^3$

   d. To divide powers of ten, subtract the exponent of the divisor (denominator) from the exponent of the dividend (numerator).

   EXAMPLES: $10^6 \div 10^2 = 10^6/10^2 = 10^{6-2} = 10^4$
   $10^3 \div 10^{-5} = 10^3/10^{-5} = 10^{3-(-5)} = 10^{3+5} = 10^8$

5. Order of Magnitude

   a. An approximation; the value of a number expressed to the nearest power of ten.

   b. Writing a number in Scientific Notation gives the order of magnitude immediately.

   c. EXAMPLES: $123 = 1.23 \times 10^2 \rightarrow 10^2$
   $0.00412 = 4.12 \times 10^{-3} \rightarrow 10^{-3}$
   $86{,}400 = 8.64 \times 10^4 \rightarrow 10^5$ in order of magnitude because 8.64×10^4 is closer to 10×10^4 than it is to 1×10^4

QUANTITATIVE SCIENCE DATE____________ NAME____________________

EXPRESS THE FOLLOWING IN SCIENTIFIC NOTATION, i.e., one digit to the left of the decimal point and the appropriate power of ten, or M X 10^n. SHOW ALL WORK ON THIS SHEET.

1. 54 5.4 X 10^1
2. 498 __________
3. 7618 __________
4. 0.622 __________
5. 0.00875 __________
6. 0.0463 __________
7. 73,170,000 __________
8. 0.00000998 __________
9. 0.000010010 __________
10. 578,000,000,000 __________
11. 387 thousand __________
12. one thousand million __________
13. 323 hundred thousandths __________
14. 7.44 billion __________

WRITE THE FOLLOWING IN THE USUAL DECIMAL FORM:

15. 3.77 X 10^2 377
16. 5.66 X 10^3 __________
17. 6.65 X 10^{-2} __________
18. 4.44 X 10^5 __________
19. 3.32 X 10^{-4} __________
20. 7.92 X 10^6 __________
21. 2.99 X 10^{-6}
22. 1112 X 10^3
23. 6.53 X 10^0
24. 367 X 10^{-3}
25. 0.0456 X 10^{-2}

26. The distance from the sun to the earth is approximately 93,000,000 miles. Express this distance in Scientific Notation.

26__________

27. The number of stars in the MILKY WAY has been estimated at 100,000 million. Express this number in Scientific Notation.

27__________

28. The average wavelength of red light is 7.10 X 10^{-5} centimeters. Express this in the usual decimal form.

28__________

29. The mass of one hydrogen atom is about 1.64 X 10^{-24} grams. Express this mass in decimal form.

29____________________

30. The X-unit is defined as 10^{-12} meters. Express this in decimal form.

30____________________

31. The diameter of a molecule of hydrogen is approximately 2.4 X 10^{-10} meters. Express this in decimal form.

31____________________

32. In the study of very short electrical pulses, the term NANOSECOND has come into use. The nanosecond is a time interval equal to one-thousandth of a millionth of a second. Express this in scientific notation.

32__________

QUANTITATIVE SCIENCE DATE________________ NAME________________________

REWRITE EACH FACTOR OF THE EXPRESSION IN SCIENTIFIC NOTATION
IN THE first COLUMN, ESTIMATE THE VALUE OF THE NUMBER PART BY SUBSTITUTING
APPROXIMATE WHOLE NUMBERS
IN THE second COLUMN, COMPUTE THE RESULT OF THE POWERS-OF-TEN PART
IN THE third COLUMN, COMBINE COLUMNS 2 and 3 TO WRITE THE ESTIMATED TOTAL RESULT
IN THE fourth COLUMN, USE SLIDE RULE AND ESTIMATED RESULT TO WRITE THE CORRECT
FINAL ANSWER

	ESTIMATE #	POWERS OF TEN	EST. TOTAL	CORRECT
0. $32 \times 17 \times 21$	12	10^3	1.2×10^4	1.14×10^4
1. 714×8660				
2. 0.00417×0.692				
3. $479 \times 974 \times 26{,}400$				
4. $0.279 \times 0.116 \times 0.342$				
5. $\frac{69 \times 178 \times 8710}{9.40 \times 71 \times 4210}$				
6. $\frac{49{,}400 \times 69{,}600 \times 92{,}900}{0.462 \times 0.694 \times 0.278}$				
7. $\frac{0.00111 \times 0.000713 \times 0.0944}{632 \times 74.1 \times 864}$				
8. $\frac{0.000111 \times 0.0924 \times 0.00333}{6{,}420{,}000 \times 2400 \times 186{,}000}$				
9. $\frac{64{,}000 \times 0.0621 \times 171{,}000}{494 \times 13 \times 48}$				
10. $\frac{45{,}600 \times 6400 \times 9310}{0.00161 \times 0.00611 \times 0.000131}$				
11. $\frac{15.3 \times 28.4 \times 17{,}800}{0.00932 \times 0.00616}$				
12. $\frac{0.423 \times 0.169 \times 0.00000688}{0.597 \times 0.00975}$				

QUANTITATIVE SCIENCE DATE__________ NAME______________________

SLIDE RULE EXERCISES; EXPRESS THE RESULT IN SCIENTIFIC NOTATION

1. $4980 \times 606 \times 478$

1__________

2. $0.000262 \times 0.00623 \times 0.01780$

2__________

3. $$\frac{6120 \times 0.895}{534}$$

3__________

4. $$\frac{0.0232 \times 19.7 \times 0.311}{98.5 \times 454}$$

4__________

5. $$\frac{7630 \times 36{,}700 \times 673{,}000}{5{,}110{,}000 \times 386{,}000}$$

5__________

6. $$\frac{0.815 \times 0.00188}{0.0442 \times 0.484 \times 0.00636}$$

6__________

7. $$\frac{640 \times 73.7 \times 9.25}{0.0955 \times 0.629 \times 0.714}$$

7__________

8. $$\frac{0.411 \times 5.87 \times 0.1094}{795 \times 7880 \times 83{,}300}$$

8__________

9. $$\frac{4820 \times 793 \times 8410}{10{,}120 \times 1102 \times 1220}$$

9__________

10. $$\frac{0.120 \times 3300 \times 234}{722 \times 682 \times 592}$$

10__________

LENGTH MEASURE

Beside providing practice in measuring in both metric and English units, this simple exercise

a. introduces via the laboratory several length standards of historical importance (#1 on LAB INSTRUCTION SHEET),

b. provides some meaningful simple slide rule problems (#1 and #4),

c. Points out the advantage of the metric over the English system,

d. confronts the student with easy problems involving length measure (#3, #4, and #5),

e. permits a quick check of earlier assignment to number each page of lab notebook! (In #4, the necessary thickness can readily be obtained by measuring the height of the stacked pages and dividing by the number of pages in the stack),

f. casually introduces precision measure (#5) and the use of finer instruments.

Incidentally, the measure of the wire's diameter with a ruler is made by coiling closely many turns of wire around some object like a pencil, dowel, or even finger. The same principle of the stack height divided by the counted number used in #4 applies here.

QUANTITATIVE SCIENCE DATE__________________ NAME__________________

LAB INSTRUCTION SHEET: Length Measure

1. Measure the following lengths in centimeters and inches and record in the space provided. Place the result of dividing the length in centimeters by the length in inches in the third column.

   a. from tip of left elbow to tip of middle finger on left hand
   b. from tip of left thumb to first joint on thumb
   c. from tip of thumb to tip of smallest finger with hand outstretched
   d. from tip of right hand to tip of left hand with arms outstretched

cm	in	cm/in

2. Measure and record the width of the corridor (in meters and in feet plus inches)

meters	ft. + in

3. Measure in centimeters only the four bricks lying flat (any side possible) on the table top, each brick in contact with at least one other brick and

a. arranged to give greatest possible length	
b. arranged to give shortest possible length	

   BE SURE TO SCRAMBLE THE BRICKS BEFORE YOU LEAVE THE TABLE!

4. Use only a ruler to obtain the dimensions necessary to compute the volume of one page of laboratory notebook; identify the dimension measured and compute the volume.

5. Use only a ruler to obtain the dimensions necessary to compute the volume of length of wire; identify the dimension measured and compute the volume. Check the measurement of small dimensions by using the MICROMETER CALIPER.

UNIT I. INTRODUCTION

E. SCIENTIFIC NOTATION (continued)

6. dex

   a. Symbol derived from "decimal exponent"

   b. Expresses the order of magnitude of the ratio of two quantities.

   c. Suggested by J.B.S. Haldane in NATURE, Volume 187, September 1960.

   d. Useful as a shorthand method of expressing the ratio of different quantities, the measuring range of an instrument, or the range of values a certain quantity may cover.

EXAMPLE: ratio of the speed of light to the speed of sound:
the speed of light is 1.86×10^5 miles/second;
the speed of sound is 1.10×10^3 feet/ second;
with both expressed in feet/second,

$$\frac{\text{speed of light}}{\text{speed of sound}} = \frac{9.81 \times 10^8}{1.10 \times 10^3} = 8.92 \times 10^5 \rightarrow 10^6 \text{ or 6 dex.}$$

EXAMPLE: range of marked divisions on a meter stick;
largest marked division is 1 meter;
smallest marked division is 1/1000 meter or 10^{-3} meter;
the ratio of the largest to the smallest marked division is 10^3 or 3 dex.

F. Length --- a FUNDAMENTAL QUANTITY

1. Defined as the measure of the interval in space between two points.

2. Early measurements

EXPERIMENT **

   a. Points up the need for standards, even in ancient times.
   b. Choice of standards emphasizes accessibility.
   c. Some early standards (compare to experimental results).

CUBIT: bent forearm, about 18 inches or 46 centimeters.
FATHOM: outstretched arms, about 6 feet.
FOOT: about 2/3 cubit.
SPAN: outstretched hand; later ½ cubit.
HAND: about 1/3 foot.
MILE: Roman origin, "millia passum", 1000 paces or about 5000 feet.

QUANTITATIVE SCIENCE DATE____________ NAME______________________

1. Use the symbol dex to express the order of magnitude of the following ratios:

a. speed of light to the speed of sound in air
1a__________

b. largest to smallest marked division on meter stick
1b__________

c. your height to the length of a grasshopper
1c__________

d. diameter of a telephone pole to diameter of a straight pin
1d__________

e. height of the TERMINAL TOWER to the height of WILT THE STILT
1e__________

f. time of one earth revolution around the sun to one earth revolution about its own axis
1f__________

g. weight of one cubic foot of water to one tablespoon of butter
1g__________

2. Give the order of magnitude (using the symbol dex) of the ratio of:

a. the driving distance (from RRHS) to downtown Cleveland to the length of the RRHS lot along Wagar Road
2a________

b. the distance from the earth to the sun to the earth's diameter
2b________

c. the temperature of an INCANDESCENT light bulb filament (on!) to the temperature of Arctic ice
2e________

3. Express symbolically the order of magnitude of

a. the range of marked divisions on a yardstick
3a__________

b. the range of marked time intervals on a clock with a sweep second hand
3b__________

c. the range of marked time intervals on a laboratory stopwatch
3c__________

QUANTITATIVE SCIENCE DATE____________ NAME______________________________

PERFORM THE OPERATIONS INDICATED. SHOW CLEARLY IN THE SPACE PROVIDED HOW THE DECIMAL POINT IS DETERMINED. EXPRESS THE RESULT IN SCIENTIFIC NOTATION. **The number in parentheses gives the point value for each problem.**

1. 1.64×4.70 (3) 1____________

2. 4.85×2.935 (3) 2____________

3. $4.48 \div 3.42$ (3) 3____________

4. $76.3 \div 198$ (3) 4____________

5. $635/0.000653$ (3) 5____________

6. $1.73 \times 4.30 \times 9.30$ (5) 6____________

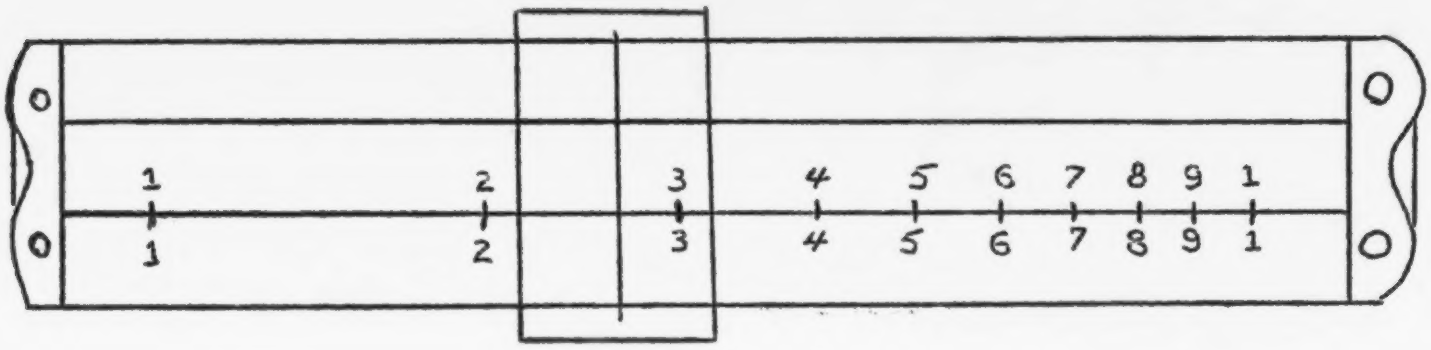
7. $\dfrac{8.95 \times 0.314}{0.209}$ (5) 7____________

8. $\dfrac{1.590 \times 36.4 \times 7.63}{4.39 \times 9.30}$ (5) 8____________

9. $\dfrac{0.000131 \times 0.00161 \times 0.00611}{45{,}600 \times 6400 \times 9310}$ (5) 9____________

10. $\dfrac{10^3 \times 10^{-2} \times 10^8 \times 10^4}{10^{-6} \times 10^5 \times 10^0}$ (5) 10____________

CLEARLY IDENTIFY ALL MAJOR FEATURES AND PARTS IN THE DIAGRAM BELOW: (10)



a____________________

b____________________

c____________________

d____________________

e____________________

f____________________

Experiments: HEIGHTS, AREAS, VOLUMES, AND SURFACE-VOLUME RATIOS

A host of simple experiments can be based on a set of cylinders, really nothing more than a rod cut into several different lengths. Wooden dowels are an obvious choice, readily available and easily cut. Lucite rod is also inexpensive, quite uniform in diameter, and not difficult to work.

The first experiment with the cylinder set (for which a student laboratory sheet is prepared) is aimed primarily at a graph of measured cylinder heights comparing metric and English measure. Later, when the straight line is studied in some detail, this graph lends emphasis to the physical significance of slope, here giving a meaningful 2.54 cm/in.

At least two additional exercises are suggested: graphing the area of the curved surfaces, again comparing metric and English measure (cm^2 vs in^2), and also the same for the volume of the cylinders (cm^3 vs in^3). The latter has proved particularly helpful when volume conversions are necessary for density determinations.

* * * * *

A set of small cubes (whose sizes were dictated by the thickness of Lucite sheet scraps available) has served as the basis for a fascinating excursion into "the size of things" via the surface-volume ratio. The cube edges were measured; the areas, volumes, and surface-volume ratios were computed. Next, the surface-volume ratio was plotted against the side length. The implications of the relationship (inverse variation) exhibited here on biological systems were discussed and for a week Haldane's article, "On Being the Right Size" (from POSSIBLE WORLDS) was the hottest thing in the library!

To exploit the simple experiment farther, the same graph can be used as an example of the hyperbola of form $y = k(1/x)$ in later graphical analysis.

QUANTITATIVE SCIENCE DATE________________ NAME________________

LAB INSTRUCTION SHEET: Cylinder Sets

1. Note that each lettered set consists of eight cylinders of different height.

2. Measure the diameter of each cylinder in centimeters using both a ruler and the micrometer. Record.

3. Measure the height of each cylinder in centimeters and in inches, using both a ruler and the VERNIER CALIPER. Record.

CYLINDER NUMBER	DIAMETER (cm)		HEIGHT (cm)		HEIGHT (in)	
	RULER	MICROMETER	RULER	VERNIER	RULER	VERNIER
Total						
Average						

4. Plot a graph from your data with the CYLINDER HEIGHTS in centimeters along the vertical axis and in inches along the horizontal axis.

5. Compare the average of the diameter measurements with a ruler to the average of the diameter measurements made with the micrometer.

F. Length --- a FUNDAMENTAL QUANTITY (continued)

3. English system

a. Based on the YARD as standard.

b. Yard now defined in terms of the STANDARD METER: one yard = 36.00/39.37 meter.

c. Common use: inch, foot, mile.

d. Inch fractions: 1/2, 1/4, 1/8, 1/16, etc., now decimalized in commercial and industrial use.

4. Metric system

EXPERIMENT **

a. Based on STANDARD METER: platinum-iridium bar in Sevres, near Paris.

b. Current definition of the standard length (problem sheet).

c. kilometer (km), centimeter (cm), millimeter (mm), micron (μ).

d. 1.00 inch = 2.54 cm (compare to lab result).

e. Useful approximation: 1 foot is about 30 cm.

5. Similar triangles

EXPERIMENT **

a. Useful in length measurement; permit extension beyond direct measure.

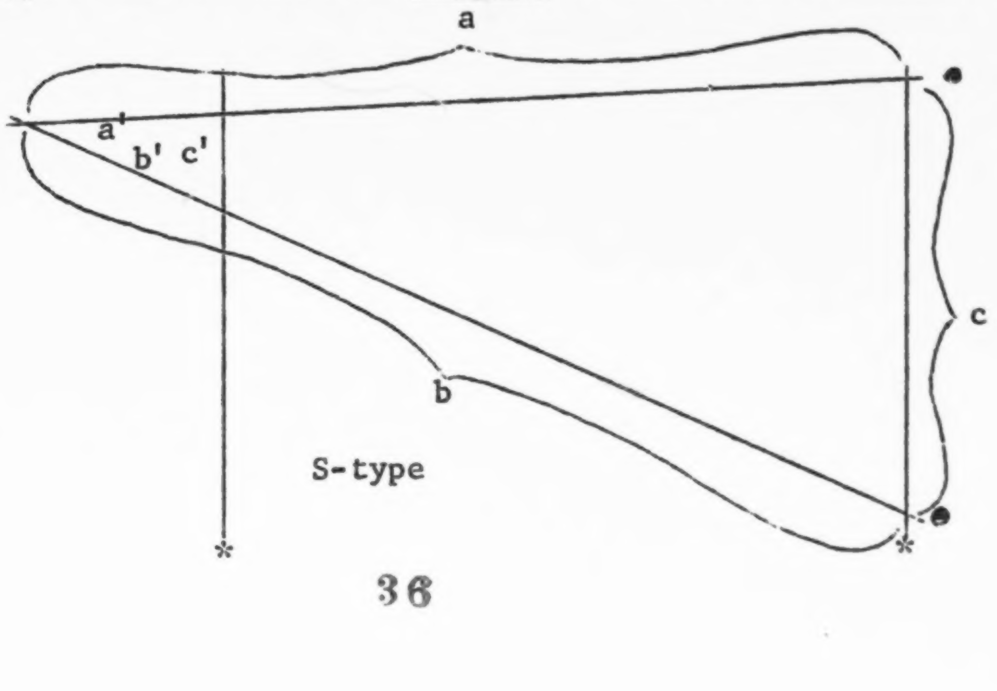
b. Will use ideas of RATIO (comparison of two quantities expressed by division) and PROPORTION (mathematical statement expressing the equality of two ratios).

c. Geometrical theorem: mathematical statement which can be (rigorously) proved; here, satisfied with "numerical proof" based on measurement rather than geometrical logic.

d. Two classes of similar triangles of interest here; referred to as S-type and P-type.

e. Both types have in common TWO PARALLEL LINES (*) crossed by TWO INTERSECTING LINES (•)

f. S-type: lines intersect outside of the parallel lines.



QUANTITATIVE SCIENCE DATE________________ NAME________________________

EXPRESS THE RATIO OF THE FIRST QUANTITY TO THE SECOND: REDUCE TO SIMPLEST TERMS AND EXPRESS DECIMALLY.

1. 16, 32 1________________

2. 2 yards, 1 foot 2________________

3. 10 minutes, 4 hours 3________________

4. 20 cents, 2 dollars 4________________

5. 15 minutes, 2 hours 5________________

6. 1 hour, 40 seconds 6________________

7. 4x inches, 8x feet 7________________

8. 15x feet, 5y yards 8________________

9. 12 megagrunts, 3 microgrunts 9________________

10. 62.5 centimeters, 15 meters 10________________

11. .0005, 3.674 11________________

12. 14a, 24b 12________________

13. $7x^2$, 21x 13________________

14. $64xy^2z^3$, 24xyz 14________________

15. 14abc, $84a^2b^2c^2$ 15________________

16. Express the ratio of the circumference of a circle to its diameter. 16________________

17. Gold marked 14K (carats) means that it consists of 14 parts of pure gold and ten parts of some other metal. How many ounces of pure gold are there in an object marked 14K which weighs 48 ounces? 17________________

18. Compare the amount of acid in 20 gallons of a 10% solution to the amount of acid in 2 gallons of a 5% acid solution. 18________________

19. A photograph is 2¼ inches wide and 3¼ inches high. When enlarged, how high will the print be if the width is to be 8 inches? 19________________

20. The scale on a map is ¼inch = 50 miles. What is the distance in miles between two cities which are 5½ inches apart on the map? 20________________

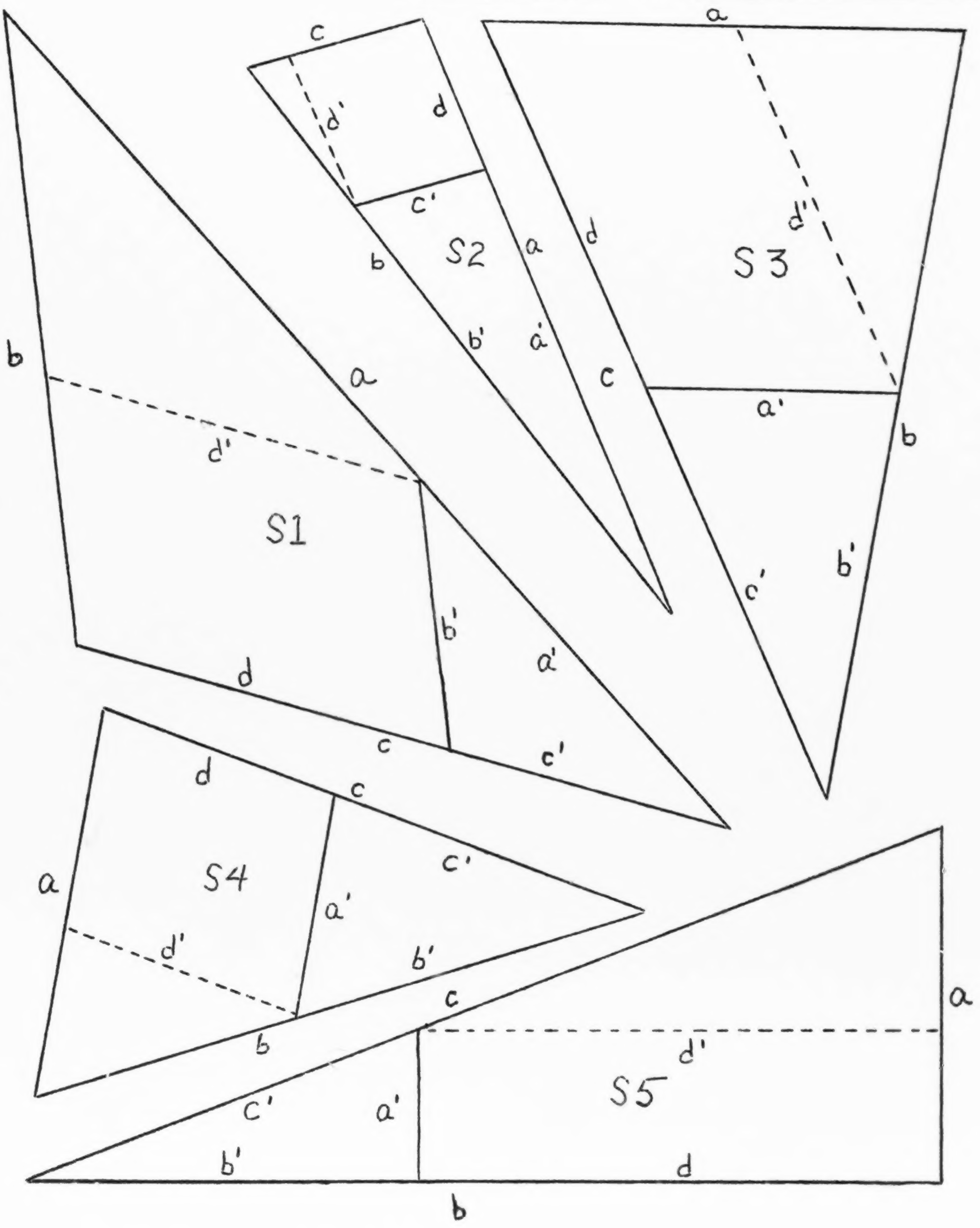
21. Circle C_1 has a radius r and circle C_2 has a radius 2r. Find the ratio of the areas of the two circles. 21________________

QUANTITATIVE SCIENCE DATE__________ NAME______________________________

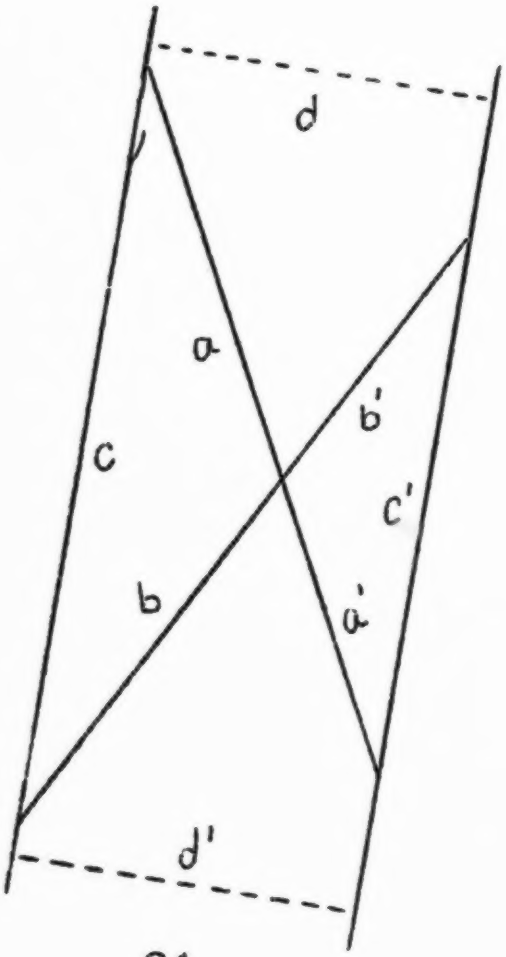
SOLVE THE FOLLOWING PROPORTIONS. SHOW YOUR WORK IN THE SPACE PROVIDED. PERFORM ALL MULTIPLICATIONS AND DIVISIONS ON THE SLIDE RULE

1. $x/5 = 5/25$ 1__________
2. $2:x :: 6:21$ 2__________
3. $x/9 = 13/0$ 3__________
4. $5/x = 5/24$ 4__________
5. $19/4 = x/8$ 5__________
6. $7:x::3:24$ 6__________
7. $20/5 = x/2$ 7__________
8. $20/3 = 2x/3$ 8__________
9. $14/18 = 14/2x$ 9__________
10. $3x/8 = 1/6$ 10__________
11. $7x/8 = 14/1$ 11__________
12. $x:12 :: 3:16$ 12__________
13. $12/5x = 2/15$ 13__________
14. $7x/9 = 14/3$ 14__________
15. $9x/21 = 45/21$ 15__________
16. $8/1 = 7x/5$ 16__________
17. $x/2 = 4/0$ 17__________
18. $2x/8 = 11/6$ 18__________
19. $17/4x = 17/34$ 19__________
20. $3:16 :: 9x:12$ 20__________
21. $1:6 :: 2:3x$ 21__________
22. $3/4 = 9x/10$ 22__________
23. $x:0 :: 15:3$ 23__________
24. $x:42 :: 0:84$ 24__________
25. $a/bx = a^2/b^2$ 25__________

DATE____________ NAME____________________


S1
b
d'
a
b'
a'
d
c
c'
S2
c
d'
d
c'
b
a
b'
a'
S3
a
d'
d
c
a'
b
b'
c'
S4
d
c
c'
a
a'
d'
b'
b
S5
c
a
d'
a'
c'
b'
d
b

DATE_______________ NAME___________________________



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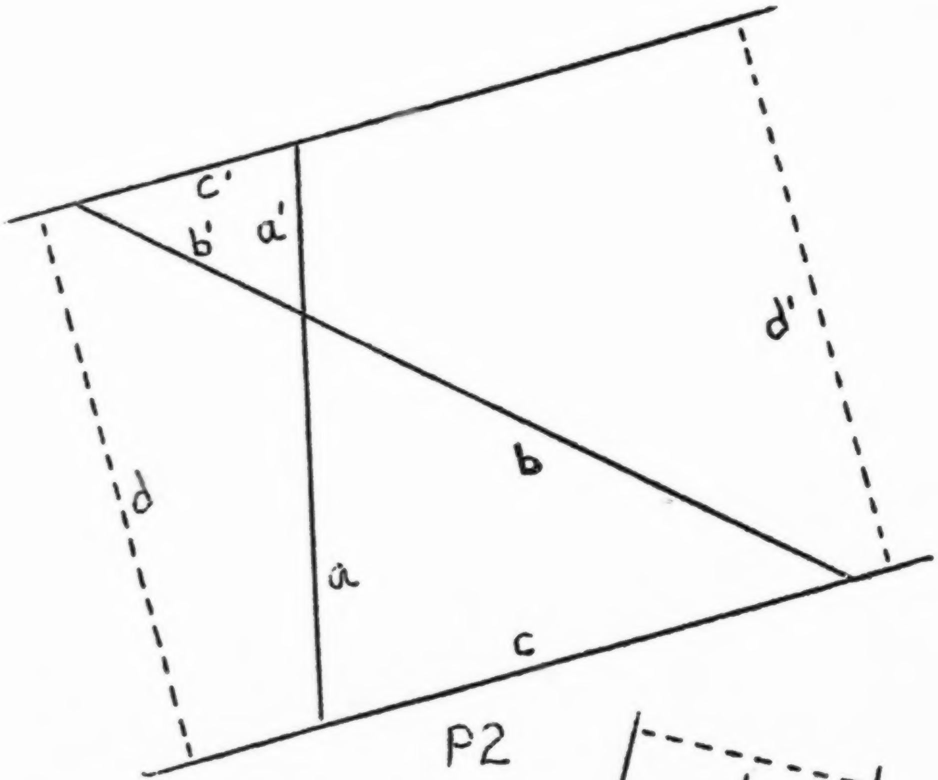

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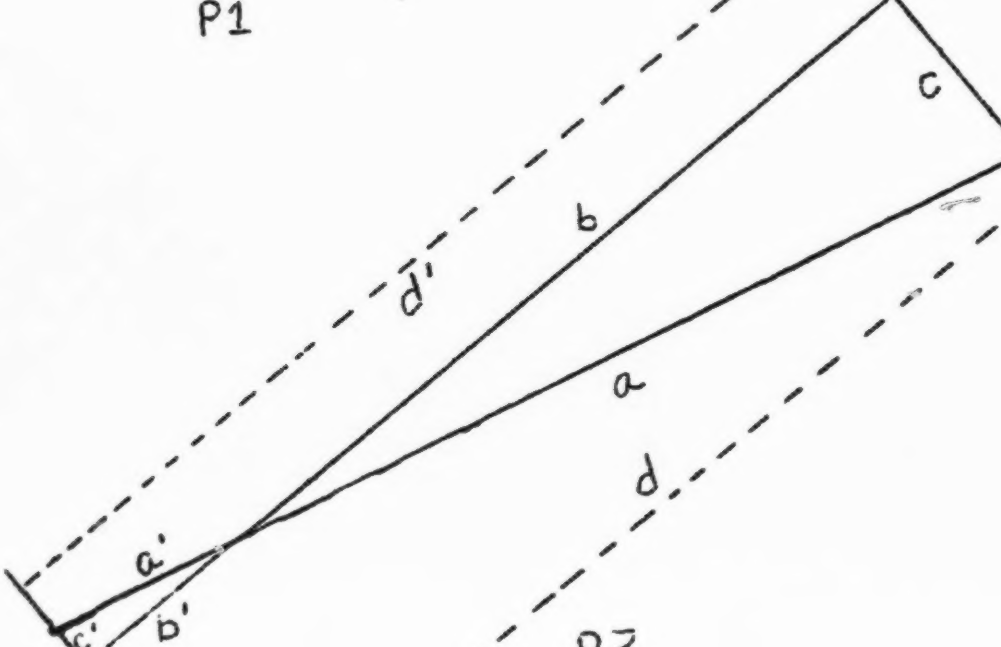
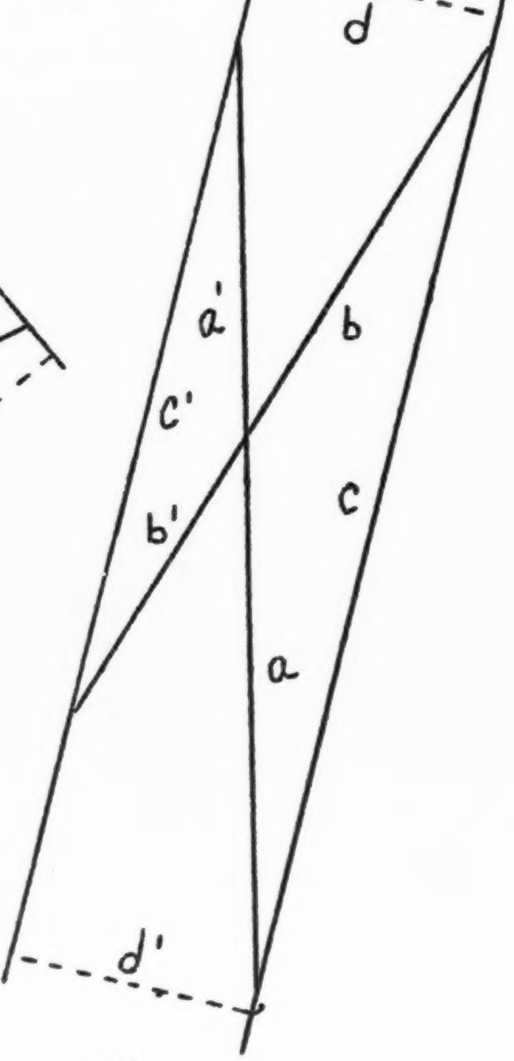


P3



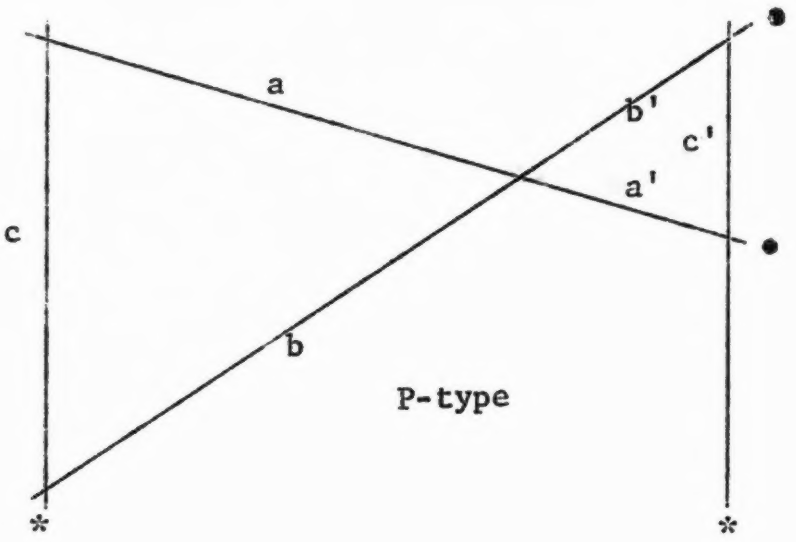
P4

UNIT I. INTRODUCTION

F. Length --- a FUNDAMENTAL QUANTITY

5. Similar triangles (continued)

g. P-type: lines intersect between the parallel lines.



h. THEOREM (based on results of "experiment"): When two parallel lines are crossed by two intersecting lines, the corresponding sides of the similar triangles formed are proportional (or the ratios of the corresponding sides are equal).

$$a/a' = b/b' = c/c' \quad \text{also} \quad a'/a = b'/b = c'c$$

i. Note carefully which sides are corresponding, particularly in P-type similar triangles.

6. The DERIVED QUANTITIES of area and volume follow immediately from the concept of length.

a. Dimensionally, area is length squared. EXPERIMENT*

b. Dimensionally, volume is length cubed. EXPERIMENT*

c. The quantity, area/volume or SURFACE-VOLUME RATIO is of great importance in living matter. EXPERIMENT*

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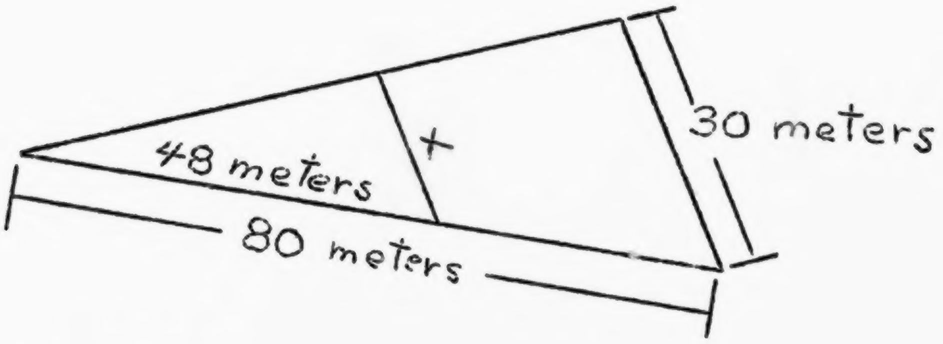
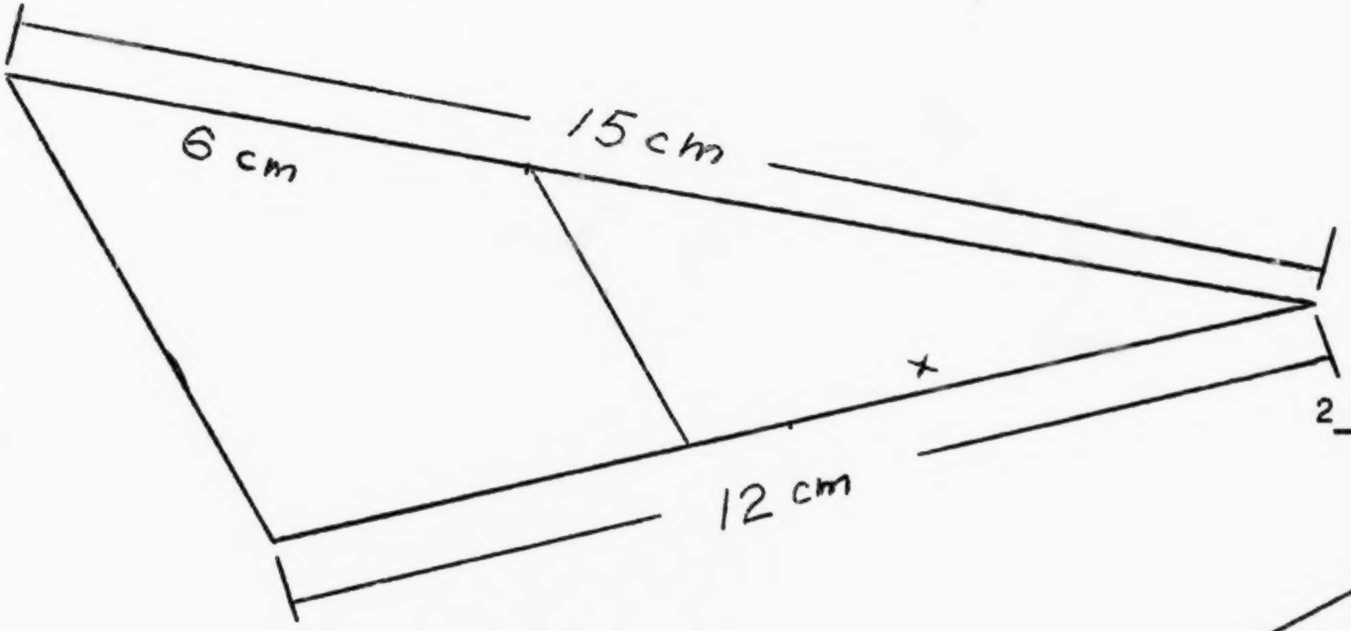
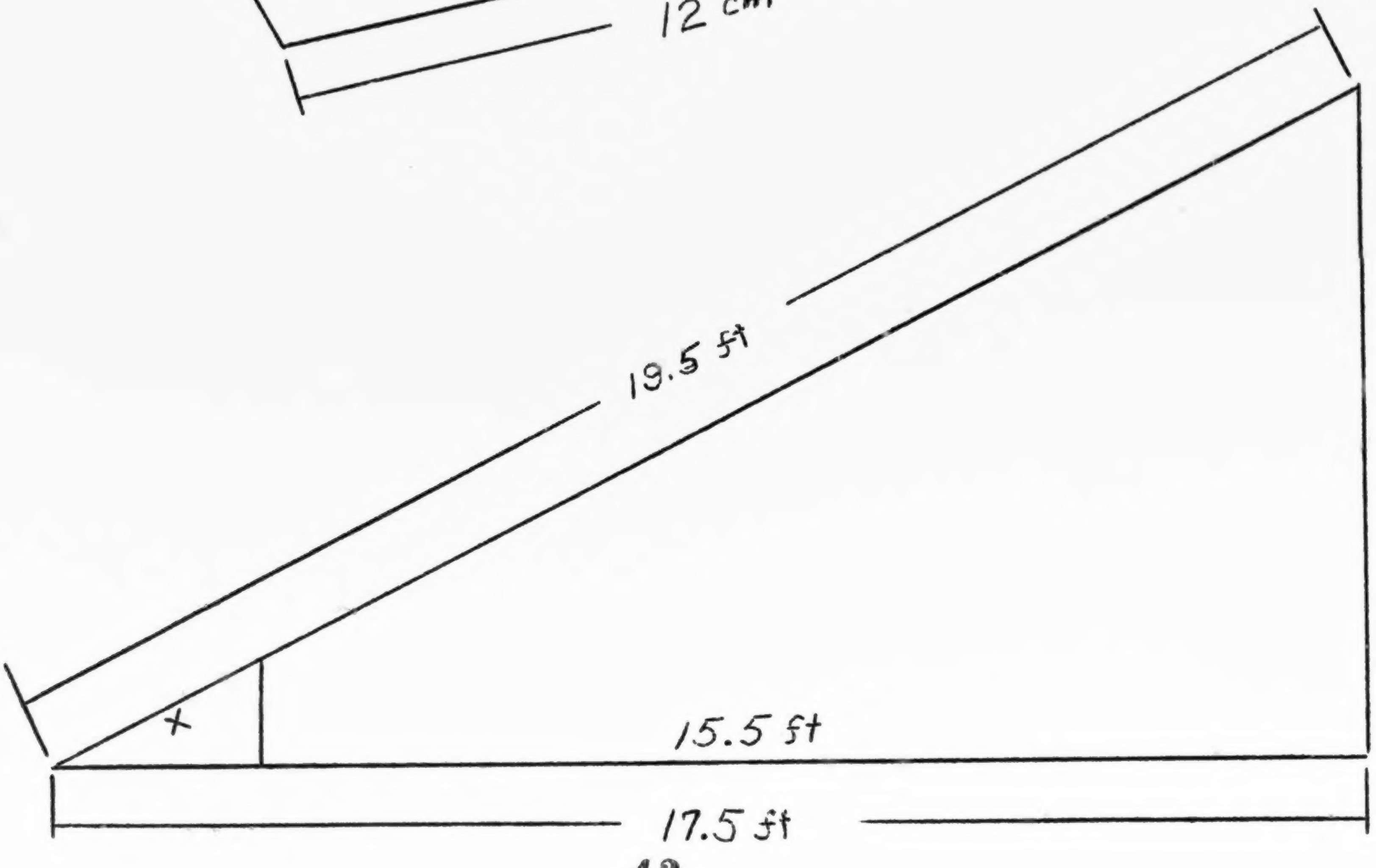
USE THE INFORMATION GIVEN TO SOLVE FOR x IN EACH CASE:



1______________



2______________



3______________

QUANTITATIVE SCIENCE **INSTRUCTOR INFORMATION SHEET**

EXPERIMENT: Measurement of Flagpole (or smokestack)
Height by Use of Sextant

1. Mark off along a line in each of (at least) three directions 10-meter intervals, starting at the base of the flagpole. Be sure to correct for the radius of the flagpole at the base, particularly if it is an appreciable fraction of a meter. Fifty- or one hundred-foot tape measures are convenient for this; 10 meters is equivalent to 32'10". Using three (or more) different lines provides a large number of measuring stations and also allows freedom to move away from a direction in which the students may be looking directly into the sun.

2. Students sight the very top of the flagpole with the sextant and thereby measure the angle from their eyes to the top of the flagpole at several (5 - 8) 10-meter intervals. A suggested data table is as follows:

Distance (meters)	Angle (degrees)	Blank	Blank	Blank

3. Be sure to elicit the information that a measurement of the sextant level from the ground is necessary; this is referred to below as the "reference height".

4. An additional measurement may be suggested. Have the sextant set at 45° and walk away from the flagpole until the top is in perfect sight. Record the distance to the flagpole. Ask: What's special about 45°?

5. IN LAB BOOK:
    a. Mark off a scaled base equivalent to the largest distance measured (50-80 meters) along the long dimension of the page.

    b. Use a protractor to reproduce angles measured at each distance station.

    c. The intersection of each angled line with the vertical drawn at zero distance (position of flagpole) gives a measured height of the flagpole above the reference height. (The vertical line must be scaled in the same way as the horizontal for this to be true). This raw flagpole height value is recorded in the first blank column.

    d. Add reference height to each raw flagpole height and record this set of measurements in the second blank column.

    e. Compute the average value of these flagpole height measurements; in the last blank column, compute the percent difference from the average value.

    f. The "correct" flagpole height may also be supplied to the students, and a per cent error could then be computed. It may be of some value to compute the average flagpole height from all the measurements made in all classes.

    g. Question: does the study of similar triangles (S-type and P-type) just completed have any relation to this experiment?

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Each reported flagpole height value obtained in the experiment with the sextant last week is listed below. Use these figures and your experimental result to do the following:

FLAGPOLE HEIGHT (in meters)

24.4
22.7
22.8
22.8
22.4
22.9
23.0
22.7
25.9
26.4
22.2
24.1
26.1
22.8
22.1
23.9
23.0
21.6
21.9
23.6
26.5
26.5

24.6
24.2
23.6
22.9
24.3
25.6
18.7
22.2
20.5
20.1
23.4
22.0
24.0
23.3
23.9
21.7
23.6
23.6
23.8
21.5
25.7
23.5

22.3
22.1
24.1
23.5
18.2
22.5
24.0
21.0
22.9
22.0
24.6
22.0
20.5
25.0
21.0
22.5
22.3
23.5
24.2
26.6
28.5
22.9
25.3

1. Calculate the average of all the flagpole height measurements given.

1____________________

2. Give your experimental result (from your lab book) for the flagpole height.

2____________________

3. Find the PERCENT DIFFERENCE between your value and the average of all the measurements given.
SHOW YOUR WORK CLEARLY.

3____________________

UNIT I. INTRODUCTION (continued)

G. Time --- a FUNDAMENTAL QUANTITY

1. Definitions
   - a. Interval between two happenings.
   - b. Measurable duration.
   - c. Period during which an event takes place.

2. Measurement of time based on a succession of repeated happenings or periodic events; EXAMPLES: heart beat, pendulum swing, spin of earth, orbit around sun.

3. INTERVAL of time vs INSTANT of time.

4. Happy standard! Universal agreement on the SECOND as the standard time unit.
   - a. Originally, the MEAN SOLAR SECOND defined as 1/86,400 of the MEAN SOLAR DAY.
   - b. But earth is irregular in its motion, so later the EPHEMERIS SECOND defined as 1/31,556,925.9747 of the year 1900.
   - c. Current definition of the time standard based on atomic vibrations (see problem sheet).

5. Repeating events: FREQUENCY and PERIOD
   - a. Frequency f is defined as the NUMBER OF COMPLETE MOTIONS PER UNIT TIME.
   - b. EXAMPLE: a 3600 rpm motor has a frequency of 3600 revolutions per minute, or preferably, 60 revolutions per second.
   - c. Frequency f is most commonly expressed in CYCLES/SECOND.
   - d. The PERIOD T is the TIME REQUIRED FOR ONE COMPLETE CYCLE OF THE MOTION.
   - e. EXAMPLE: the 3600-rpm motor mentioned above completes 60 revolutions in each second, so each revolution takes 1/60 of a second; T = 0.0167 seconds.
   - f. In general, for PERIODIC MOTION, the PERIOD T and the FREQUENCY f are reciprocally related, that is, $\boxed{T = 1/f}$, something that every educated person should understand and remember!
   - g. EXAMPLE: radio station WIXY broadcasts a radio signal at the assigned frequency of 1260 kcps; since a kilocycle is 10^3 cycles, the frequency f is 1.260×10^6 cycles/second; the period T is then 1/f, which gives 7.94×10^{-7} seconds (incidentally, this gives the ORDER OF MAGNITUDE of the time for one cycle of an AM radio signal as 10^{-6} seconds, or 1 microsecond).

6. Laboratory: PULSE and TIME. EXPERIMENT *

7. Laboratory: SIMPLE PENDULUM EXPERIMENT *

Experiment: PULSE and TIME

For this experiment classroom timers, stopwatches, or wristwatches equipped with a second hand may be used. Students working in pairs are instructed to measure and record the time required for one individual heartbeat; then two heartbeats in succession, then three, five, ten, twenty, and finally forty or some other large number. The results are to be displayed graphically: time (seconds) vs number of heartbeats.

The principal value of this experiment lies in the realization that the measurement of the short time interval required for one (or a very few) heartbeat involves relatively large inaccuracies. The consistency of the measurements for ten or more heartbeats (all these points fall nicely on a line) points to a practical conclusion: for measuring the period of repeating motion (like the pendulum to be investigated next), the measurement of the time required for a large number of motions divided by the number of motions is a far better result than the measurement of a single motion. This is an excellent point of departure for discussion of PERCENT ERROR.

A study of the pattern of recovery after rapid physical exercise is also interesting and informative. A student either jumps in place or runs up stairs (of course, WHAT and HOW MUCH must be recorded) and then has his pulse monitored until the normal rate is restored. Recording and plotting the elapsed time at 20-heartbeat intervals provides a graphic description of the recovery. The meaning of recovery rate is immediately evident and readily admits of further discussion.

An excellent, novel, additional related activity is suggested: "Demonstrating Bradycardia in a Classroom", is described by Jack Friedman in the January 1967 issue of SCIENCE TEACHER'S WORKSHOP, Parker Publishing Company, West Nyack, New York.

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1. Use periodicals and other recent source materials to find the CURRENT definitions of the standards of

   a. LENGTH

   b. TIME

   Be sure to cite the sources of your information

2. What is the period of vibration of a harp string sounding the note "low C" ?

3. a. Calculate the order of magnitude of your age in seconds at the end of this calendar year.
   b. Express symbolically (using "dex") the order of magnitude of the ratio of (one) parent's age to yours at the end of this calendar year.

4. Find the order of magnitude of the time it takes for one vibration of an FM radio wave.

5. The most common signal observed in radio astronomy is believed to originate in distant galaxies and is known to have a period of vibration of .704 nanoseconds. Express the frequency of this radiation in Gcps, where G is a symbol for the metric prefix "giga", equivalent to a factor of 10^9.

6. Use the result of the experiment on pulse and time to calculate the number of times your heart will beat

   a. During one after-game canteen

   b. While you are a student at RRHS

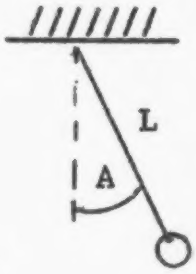
QUANTITATIVE SCIENCE
EXPERIMENT: The Simple Pendulum

INSTRUCTOR INFORMATION SHEET
(In my opinion, this particular investigation epitomizes the spirit of intelligent, quantitative, guided inquiry - LJB)

1. Setting up the PROBLEM:

   a. bob of mass m and radius r hangs on a light cord of length L; cord is attached to fixed support, bob is free to swing (confine to plane), and does so when displaced on angle A from vertical.

   b. Galileo and Sanctuary Lamp -
      each swing of lamp took same time: Periodic Motion
      (ten swings take ten times as long as one swing)
      DURING THE EXPERIMENT, YOU MAY ASSUME THE VALIDITY OF GALILEO'S STATEMENT but must verify its correctness before experiment is complete.



   c. Period of pendulum T (sec): Time required for one complete swing.

   d. Experiment must answer:
      What factors influence the period of the pendulum?
      On what factors does the period of the pendulum depend?
      On what factors does it not depend?

      Investigate the features of pendulum to determine which do and which do not affect the period of motion. Example: A, m, r, L, etc.

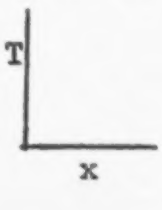
2. Investigate one factor at a time: control all others.

   Systematically: eliminate non-influencing factors,
   discover influential factors,
   study each (good range of data)

3. Determinations: are all quantitative -- measure in cgs system.

   Example: measure angle A with protractor; m with balance; L with meter stick. Express A in degrees, m in grams, L in centimeters, etc.

4. Express result of investigating each factor on a separate graph.
   (several graphs on one page of notebook)



where x is the particular factor being investigated.

5. (In groups of three) Plan the experiment before proceeding.
   a. Which factors may affect the period? Name them.
   b. Procedure on how these will be investigated. CONTROL.
   c. Apparatus needed. "Ask and you shall receive" - - - - - - -

(string-various
(bobs-various
(timer
(clamps

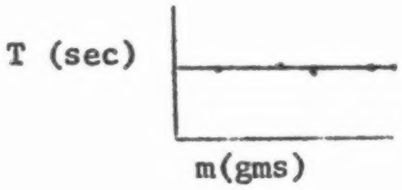
6. Laboratory -- Experimental Problem ---- forbidden to use library, textbooks, (or fathers!) for reference.

Progress Report on Pendulum Experiment.

(After the students have spent some time both floundering and progressing, ((we allow 2-3 class periods)), their T vs x graphs are checked and the following is introduced in class discussion.

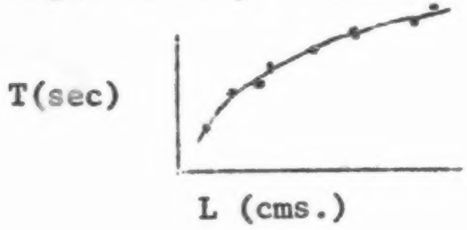
1. LIST ALL FACTORS INVESTIGATED (by everyone):
   - mass of bob
   - diameter of bob
   - angle of displacement
   - thickness of string
   - length of cord
   - etc.

2. Graphs:



straight line parallel to horizontal axis

similar for diameter of bob, string thickness, angle of displacement, etc., BUT NOT FOR LENGTH:



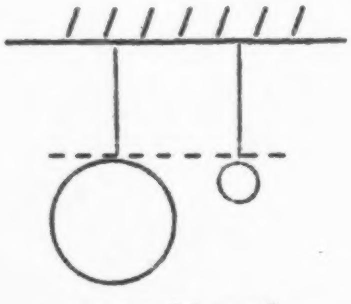
3. T most profoundly influenced by L!

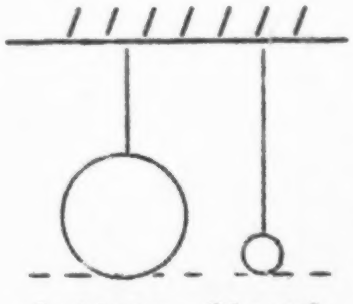
   a. investigate this closely and carefully: at least 20 points on graph; 10 of these for values of L 20 cm.; 5 for $20 \lesssim L \lesssim 40$
   5 for $40 < L \lesssim 100$

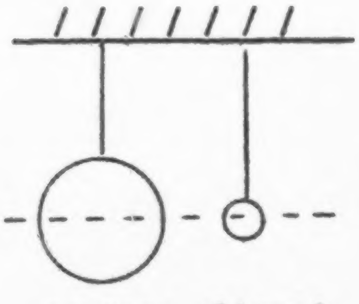
   b. what precisely is this "length of pendulum"?
   to top of bob? to bottom of bob? to center of bob?
   some other point on bob?
   answer this question experimentally.

   HINT: consider hanging a bowling ball and a BB as pendulum bobs side by side. Start the two into motion at the same time and observe their comparative swings:

   with the tops of the bobs aligned.
   with the bottoms of the bobs aligned.
   with the centers of the bobs aligned.


tops aligned


bottoms aligned


centers aligned

(The effect will be most obvious if the string is short. Why?)

Progress Report on Pendulum Experiment (con'd)

*(if time permits) closer look at dependence of period on angular displacement

a. large number of points for angles greater than 30°

b. calculate the average % change in T per degree

c. compare to the average % change in T per cm. from length data

4. Problem sheet: after completing, trim and mount in Lab Book.

Leave two pages (left and right) blank in Lab Book before beginning next experiment. This is for a later analysis of the graph.

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FROM YOUR GRAPH OF THE PERIOD T (seconds) vs THE LENGTH L (centimeters) IN THE EXPERIMENT ON THE SIMPLE PENDULUM:

1. What length corresponds to a period of
    a. 0.5 sec. 1a ______________
    b. 1.0 sec. 1b ______________
    c. 1.5 sec. 1c ______________
    d. 2.0 sec. 1d ______________

2. What is the period when the length is
    a. shortest measured 2a ______________
    b. 10.0 cms 2b ______________
    c. 15.0 cms 2c ______________
    d. 45.0 cms 2d ______________
    e. 75.0 cms 2e ______________

3. a. L_1 = 25.0 cms; L_2 = 100 cms
    T_1 = ____ sec; T_2 = ___ sec

    b. L_1 = 16.0 cms; L_2 = 64.0 cms
    T_1 = ____ sec; T_2 = ____ sec

    c. L_1 = 12.0 cms; L_2 = 48.0 cms
    T_1 = ____ sec; T_2 = ____ sec

    d. L_1 = 20.0 cms; L_2 = 80.0 cms
    T_1 = ____ sec; T_2 = ____ sec

    e. from a, b, c, d, above; when the length is quadrupled,

    the period correspondingly ________________________.

4. a. T_1 = 0.60 sec; T_2 = 1.80 sec
    L_1 = ____ cms; L_2 = ____ cms

    b. T_1 = 1.98 sec; T_2 = 0.66 sec
    L_1 = ____ cms; L_2 = ____ cms

    c. from a, b, above: for the period to change by a factor of three, the length must change by a factor of ________________________.

5. What kind of relation between the period and the length is suggested by the results of 3 and 4 above?

____

____

____

____

____

H. Mass --- a FUNDAMENTAL QUANTITY

1. Definition
    a. Quantitative measure (numerical amount) of an object's inertia.
    b. Inertia: resistance of an object to a CHANGE in its STATE OF MOTION.
    c. Mass is a measure of the reluctance of an object to be set into motion if it is at rest, or to have its motion changed if it is already moving.
    d. EXAMPLE: three balls on lawn, identical in outward appearance, size, shape, color, etc.; give each same hard kick in order; FEEL the ease or difficulty in changing the state of motion of a styrofoam ball, a wood ball, and a lead ball, respectively!

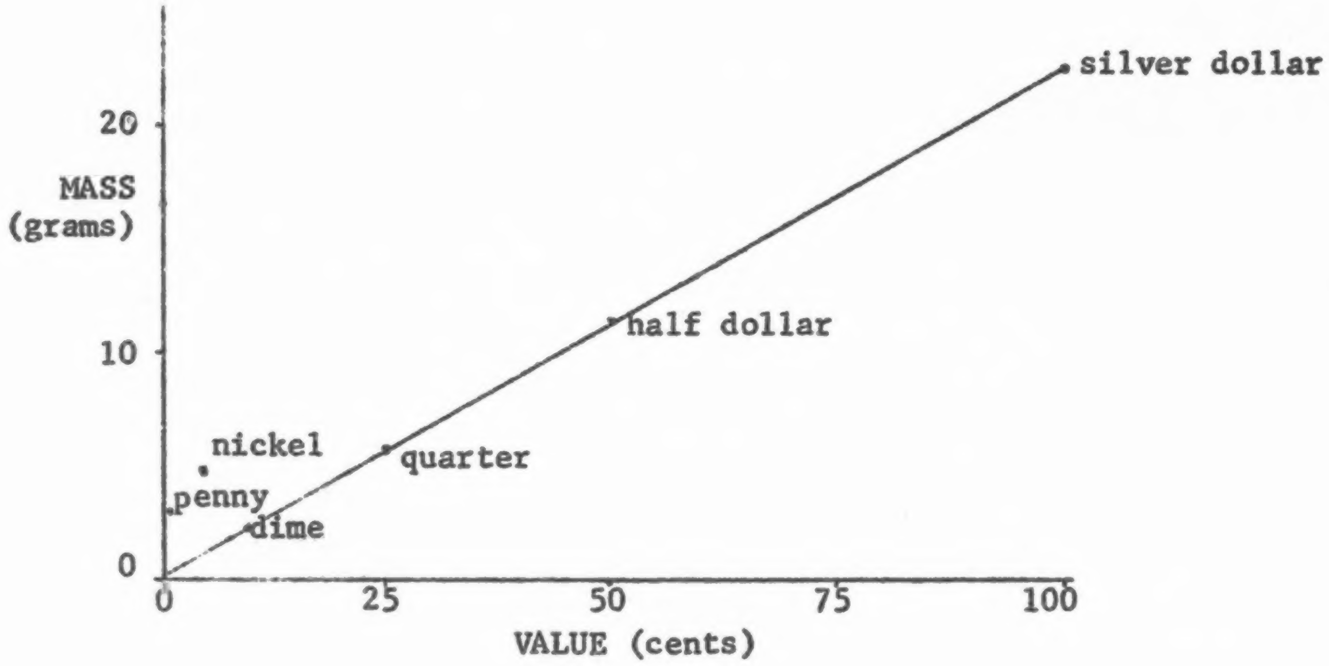
2. Mass does not depend on location; mass is the SAME on the earth, in the earth, on the moon, near the sun, stars, in free space, too.
    a. Mass can be determined entirely independently of gravity (whatever that is!).
    b. Desire a method for measuring mass that does not depend on the earth (or anything else); a method that can be used on the earth, in orbit, in space, and give the same, consistent result.
    c. The Inertial Balance or MASSER. DEMONSTRATION
    d. Define and name an arbitrary unit of inertia or mass (girls' names are always good for a laugh); show that mass can be determined by the MASSER and identified with the TIME required for one cycle of the MASSER.
    e. Emphasize that the action of the Inertial Balance does not require or use the presence of the earth and hence gravity.

3. MASS AND WEIGHT ARE NOT THE SAME.
    a. Weight depends on the pres ence of the earth (or some other large object).
    b. Weight is an acquired property measuring the effect of something else on the object; weight is a pull or force due to another body.
    c. Weight is a local condition; weight is different in different places.
    d. An object can leave its weight here on earth but it must take its mass with it!

4. Mass is frequently (and usually) determined indirectly by making use of the property of weight: the laboratory balance.

Experiment: U. S. COINS

A quantitative excursion into mass and money can prove both facinating and informative. It takes only a few minutes for the students to obtain the necessary data for a graph of MASS (grams) vs VALUE (cents) of U.S. coins.



The graph itself is provocative and the resulting possibilities for questions, problems, and discussion are practically limitless, although not necessarily terribly scientific!

QUANTITATIVE SCIENCE DATE________________ NAME__________________
LAB INSTRUCTION SHEET

One means of identifying materials is by a property known as the MASS DENSITY (often called simply DENSITY). The MASS DENSITY is defined as mass per unit volume, or for an object whose mass is m and whose volume is V, the MASS DENSITY D is given by

$$D = m/V.$$

The MASS DENSITIES of many materials are accurately known and tabulated. These values will be referred to as D_s, or standard values of the density. The PERCENT DIFFERENCE between your experimental values of the MASS DENSITY D and the standard values D_s is given by

$$\text{PERCENT DIFFERENCE} = \frac{|D - D_s|}{D_s} \times 100\%,$$

where $|D - D_s|$ is the absolute value of the difference.

OBJECT	MATERIAL	m (grams)	V (cm^3)	D (gms/cm^3)	D_s (gms/cm^3)	% DIFF.
A -						
B -						
C -						
D -						
E -						
50 ml of H_2O						
wire						
penny						
coin						
irregular object						

H. Mass --- a FUNDAMENTAL QUANTITY (continued)

5. STANDARD KILOGRAM
   - a. Originally based on water: mass of a volume of water contained in a cube 1/10 meter on an edge at a specified temperature.
   - b. Now defined as the mass of an equivalent platinum-iridium cylinder.
   - c. The GRAM is a commonly used unit of mass.
6. The unit of mass in the English system is the SLUG. DEMONSTRATION
   - a. A mass of one slug has a weight of 32.2 pounds.
   - b. The pound is a unit of weight and NOT a unit of mass.
   - c. 1.00 slug = 14.6 kilograms; note that this is an equation of MASSES, both the slug and the kilogram are units of MASS.
7. Laboratory: mass density. EXPERIMENT **
8. Laboratory: mass vs diameter of cylinders. EXPERIMENT **

I. Units and conversions

1. Physical quantities measured are expressed with UNITS or DIMENSIONS.
   - a. UNIT: label, name, tag, dimension, specifying the kind of quantity.
   - b. Unit or dimension specifies the STANDARD to which comparison is made.
   - c. Units are an essential part of a physical quantity; a physical quantity is incomplete unless the units of measurement are given.
2. Physical quantity consists of two parts.
   - a. NUMBER part: magnitude or numerical ratio comparing the amount to a standard.
   - b. UNIT or DIMENSION part: specifies the particular standard
3. Treat units as algebraic quantities.
   - a. Perform the same arithmetic operations with units as with numbers.
   - b. EXAMPLE: in finding the area of a rectangle 8 in. by 7 in.; multiply 8 in X 7 in = 56 in^2
   - c. EXAMPLE: in finding the volume of a cube 10 cm on a side; multiply 10 cm X 10 cm X 10 cm = $10^3\ \text{cm}^3$
   - d. EXAMPLE: in finding the mass density of a sample whose mass is 97 grams and whose volume is 42 cm^3; divide 97 grams by 42 cm^3 for a mass density of 2.3 grams/cm^3
   - e. EXAMPLE: length, 2.54 cm = 1.00 in; divide both sides by 1.00 in; $\frac{2.54\ \text{cm}}{1.00\ \text{in}} = \frac{1.00\ \text{in}}{1.00\ \text{in}}$; $\frac{2.54\ \text{cm}}{1.00} = 1.00$, a PURE NUMBER, no units

     (Incidentally, 2.54 cm/1.00 in = 1.00, is the ONLY length coversion necessary in going from English to metric or metric to English units; KNOW IT!)

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LAB INSTRUCTION SHEET: Cylinders, again!

1. Note that each set consists of six right cylinders of the same material, and the same height, but of different diameter. Be sure to identify and record the cylinder material. Before proceeding with any measurements, it is important that you read the remainder of this instruction sheet so that you may properply plan your data tables.

2. Measure and record the mass of each cylinder in grams.

3. Measure and record the diameter and the height of each cylinder in centimeters.

4. Find the average of the cylinder height; record.

5. Compute the volume of each cylinder in metric units; record.

6. Construct a graph of the cylinder MASS m (in grams) along the vertical axis versus the DIAMETER d (in centimeters) along the horizontal axis.

7. Leave at least one blank page in your laboratory notebook for (later) analysis of data.

UNIT I. INTRODUCTION

I. Units and Conversion (continued)

4. Conversion factors.

   a. Arise from the existence of more than one system of units and the use of multiple and fractional units within a system.

   b. The units of the three FUNDAMENTAL QUANTITIES of MASS, LENGTH, and TIME are different in different systems; the most common systems of units are given in the following table:

SYSTEM	LENGTH	MASS	TIME
MKS (metric)	meter	kilogram	second
cgs (metric)	centimeter	gram	second
English	foot	slug	second

   c. Conversion factors offer a convenient and consistent means of changing from one system to another.

5. Valid conversion is equivalent to multiplying or dividing by one.

   a. Multiplication or division by one (the unity operator) introduces no change in the value of a quantity.

   b. Express all conversion factors in a form such that they are of UNIT VALUE.

   c. EXAMPLES: 5.28×10^3 feet = 1.00 mile; dividing both sides by 1.00 mile, $\frac{5.28 \times 10^3 \text{ feet}}{1.00 \text{ mile}} = 1.00$, so the conversion factor is 5.28×10^3 feet/mile.

   60 seconds = 1.0 minute; $\frac{60 \text{ seconds}}{1.0 \text{ minute}} = 1.0$, so the conversion factor is 60 seconds/minute.

   14.6 kilograms = 1.00 slug; conversion factor is 14.6 kilograms/slug.

   d. Note that the reciprocals of each conversion factor are also of unit value.

   e. EXAMPLES: $\frac{1.00 \text{ mile}}{5.28 \times 10^3 \text{ feet}} = 1.00$; $\frac{1.0 \text{ minute}}{60 \text{ seconds}} = 1.0$; $\frac{1.00 \text{ slug}}{14.6 \text{ kilograms}} = 1.00$

6. EXAMPLES on explicit use of conversion factors.

   a. The length of a pencil is 5.72 inches; express this length in centimeters;
   5.72 ~~in~~ X (2.54 cm/~~in~~) = 14.5 cm

   b. Convert one day to seconds;
   1.00 ~~day~~ X (24.0 ~~hrs/day~~) X (60.0 ~~min~~/~~hr~~) X (60.0 sec/~~min~~) = 8.64×10^4 sec

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CLEARLY SHOW ALL CONVERSIONS IN THE SPACE PROVIDED BY EACH PROBLEM

1. Calculate the height of the Terminal Tower in meters.

1.________________

2. Express the elevation of Cleveland above sea level in meters.

2________________

3. Convert miles to meters.

3________________

4. Express one-thousandth of an inch in millimeters.

4________________

5. Convert microns to inches.

5________________

6. Express 55 kilometers per hour in miles per hour.

6________________

7. The speed of light in air is approximately 186,000 miles per second. Express this in meters per second.

7________________

8. The local acceleration due to gravity is 32.2 feet/sec^2. Express this in meters/sec^2 and centimeters/sec^2.

8________________

9. Express 60 miles per hour in kilometers per second.

9________________

QUANTITATIVE SCIENCE DATE______________ NAME__________________________

CLEARLY SHOW ALL CONVERSIONS IN THE SPACE PROVIDED BY EACH PROBLEM

1. The speed of sound in air (at 20°C) is 331 meters per second.
   a. Express this in feet per second

1a________________

   b. Express this in miles per hour

1b________________

2. Express each of the speeds below in feet/second. THEN GRAPH feet/second vs miles/hour ON A FULL SHEET OF GRAPH PAPER.
   a. 10 mph

2a________________

   b. 30 mph

2b________________

   c. 50 mph

2c________________

   d. 70 mph

2d________________

   e. 100 mph

2e________________

3. Express the mass of the standard kilogram in slugs.

3________________

4. The MASS DENSITY of salt water is about 2 slugs per cubit foot. Express this mass density in
   a. cgs units

4a________________

   b. MKS units

4b________________

5. The mass of the earth is about 6 X 10^{24} kilograms. The mass of a copper penny is about 220 microslugs. What is the monetary value of a mass of pennies equal to the mass of the earth?

5________________

QUANTITATIVE SCIENCE DATE__________ NAME______________________

1. SELECT THE ITEM FROM THE LIST THAT BEST FITS THE STATEMENT:

LIST

A. amplitude
B. microsecond
C. inertia
D. sodium
E. laser
F. period
G. hour
H. weight
I. slug
J. frequency
K. krypton
L. second
M. mass density
N. time
O. pound
P. cesium
Q. mass
R. dyne
S. liter
T. Teflon

STATEMENT

1. mass per unit volume 1______
2. number of complete motions per unit time 2______
3. resistance to motion or change in motion 3______
4. 1.158 X 10^{-5} part of one day 4______
5. English unit of weight 5______
6. interval between two events 6______
7. quantitative measure of inertia 7______
8. time required for one complete motion 8______
9. local property of object due to presence of earth 9___
10. substance whose atomic properties are currently used to define the standard of time 10______

ORGANIZE AND SHOW YOUR WORK IN THE SPACE PROVIDED BY EACH PROBLEM:

11. If the mass density of pure aluminum is 2.7 grams/cm^3, the mass of a cube of aluminum 2.0 cm on a side is

11__________

12. In the pulse experiment, a student measures 50 seconds as the time required for 60 heartbeats; the corresponding frequency is

12__________

13. One microgram expressed in kilograms, using scientific notation is

13__________

14. A laboratory centrifuge whirls a sample at a rate of 12,000 rpm; the period of this motion is

14__________

15. One kilogram expressed in English mass units is

15__________

16. The mass of a brick is 0.155 slugs; its corresponding weight is

16______

17. A cylindrical graphite rod measures 1.0 inch in height, 2.0 inches in diameter, and has a mass of 100 grams. Find the mass density of this graphite expressed in cgs units.

17________________

18. At moderate temperatures, sound is transmitted at a rate of 3270 miles/hour in water. Express this speed in meters/second.

18________________

QUANTITATIVE SCIENCE DATE__________ NAME______________________

1. SELECT THE ITEM FROM THE LIST THAT BEST FITS THE STATEMENT:

LIST

A nanosecond
B kilogram
C pound
D dex
E length
F minute
G yard
H 1000 paces
I micron
J area
K second
L Angstrom
M foot
N inertia
O weight
P slug
Q time
R mass
S measurement

STATEMENT

1a. Interval in space between two points 1a________
1b. One hundredth of a millionth of a centimeter 1b________
1c. Resistance to motion 1c________
1d. Local property of an object due to the presence of the earth 1d________
1e. Interval between two events 1e________
1f. English unit of weight 1f________
1g. 1.158 X 10^{-5} part of one day 1g________
1h. Numerical measure of inertia 1h________
1i. Comparison with chosen standard 1i________
1j. Symbolic expression for the order of magnitude of a ratio 1j________

2. The three fundamental quantities are 2a______________

2b______________

2c______________

3. Ideal standards, although arbitrarily chosen, should have the qualities of being 3a______________

3b______________

3c______________

4. One microgram expressed in kilograms, using scientific notation is 4______________

5. The general (symbolic) expression for a number in scientific notation is 5______________

6. A certain length measure is given as 0.078 centigrunts. Express this length in grunts, using scientific notation 6______________

7. The mass of 1.0 kilogram expressed in slugs 7______________

8. Discuss the QUANTITATIVE aspect of this course in the space below:

QUANTITATIVE SCIENCE DATE______________ NAME__________________________

BE SURE TO SHOW YOUR WORK IN THE SPACE PROVIDED BY EACH PROBLEM

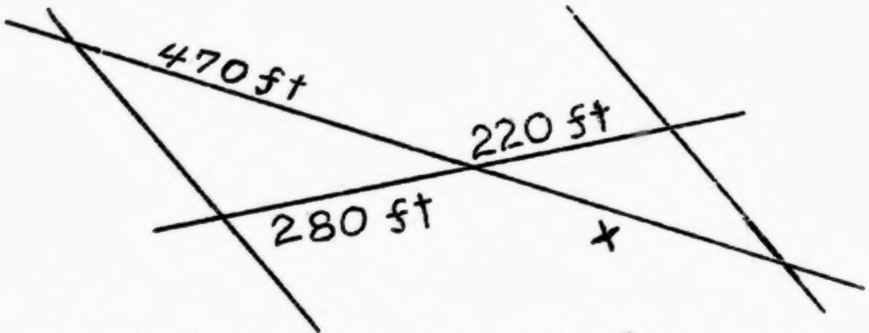
9. Find the ORDER OF MAGNITUDE of the 440-yard run expressed in CUBITS.

9__________

10. EXPRESS SYMBOLICALLY the order of magnitude of the ratio of the LENGTH of this room to the DIAMETER of a pencil.

10__________

11. SOLVE for x:



11__________

12. Sam Sextant noted a reading of 30° to the top of the smokestack measured at a distance 87 meters from the smokestack base. If his sextant was mounted on a tripod 1.0 meter off the ground, find the height of the smokestack. Draw a scale diagram.

12__________

13. A good laboratory centrifuge whirls a sample at the rate of 11,500 rpm. Find the PERIOD of motion.

13__________

UNIT II. INTRODUCTION TO OPTICS (or LIGHT)

A. Definition of light

1. Words and word roots associated with light:
    - a. from Greek: ops --- eye
      optikos --- sight, vision
      phos --- light
      leukos --- white
    - b. from Latin: lux --- light
      lucere --- to shine
      lumen --- light
    - c. Old English: liht, light
2. What is light?
    - a. For starting point, rely on everyday, common, experiences for a definition.
    - b. Suggest a sense definition as the first way to define; observations are based on our senses.
    - c. Discuss sense, operational, and theoretical definitions; three ways to define. **DEMONSTRATION**
    - d. LIGHT IS THAT WHICH AFFECTS THE SENSE OF SIGHT.
    - e. Note the limitation of any sense definition; emphasize limitations of this sense defintion of light.
    - f. Point out that this definition may be modified extended, altered, as new knowledge is acquired.
    - g. At the conclusion of the study of light in this course (and again later at conclusion of physics course) students will be asked to define light based on what they will have learned about its behavior.

B. Importance of light: knowledge and vision

1. Most of the information that reaches us --- most of the knowledge we possess --- comes to us through the sense of sight.
2. Most of our measurements in one way or another depend on light.
3. Sharpness of vision: Snellen Ratio
    - a. Clinical method originated by Herman Snellen (1834 - 1908), a Dutch ophthalmologist, to determine sharpness of vision or visual acuity. **EXPERIMENT ***
    - b. Quantitative basis: the NORMAL EYE distinguishes two points (or lines) 1/16 inch apart from a distance of 20 feet.
    - c. If the separation between points is called δ (delta), and the distance to the eye d, then for normal vision: δ = 1/16 inch
      d = 20 feet

QUANTITATIVE SCIENCE
Experiment: SHARPNESS OF VISION

INSTRUCTOR INFORMATION SHEET

This exercise provides an opportunity for the students to employ finer measuring techniques and instruments. Measuring Comparators (such as those available from FINESCALE COMPANY, Los Angeles, California, or MICROSCALE LTD., Morton Grove, Illinois, or EDMUND SCIENTIFIC, Barrington, New Jersey) as well as microscopes are used to measure the spacing between parts of letters in fine print. Any available microscope can be converted to a very precise measuring instrument quickly and conveniently with the aid of inexpensive Ronchi Rulings (EDMUND SCIENTIFIC). If a calibrated stage drive for a microscope is available, this method of measuring can also be introduced.

In essence the experiment is conducted as follows:

1. Students are asked to bring in three print samples, each of different size letters, in the range from very fine print to larger-than-average book or magazine letters. Point out that effort should be made to match the print style, that is each sample in the group of three should exhibit the same basic letter construction.

2. The δ (or spacing between parts of letters) of each print sample is measured. Consistency in measuring the same letters in each sample is important (in capital letters, "E" is excellent; avoid lower case "a", "e", and "s" --- their sometimes unusual construction yields a δ not typical of the print size). These measurements are quite appropriately made in decimal-fractional inches; the Ronchi Rulings, Clinic Eye Chart, and Snellen Ratio all use inches (and feet).

3. Next the student determines the d (farthest print-to-eye distance at which the eye resolves letters of a given δ) for each eye and for each print size. This is done by covering one eye with a card and walking from a distance toward the print sample mounted at eye level on a wall, window, cabinet, etc. (Experience has shown that walking away from the sample prejudices the results).

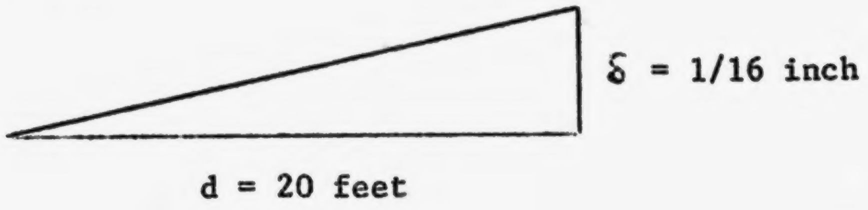
4. Each eye is also tested in the usual way with the Clinic Eye Chart, (WELCH SCIENTIFIC, Skokie, Illinois) mounted 20 feet away. The δ of the last line read completely and correctly is measured.

5. Each student now has accumulated two sets (one for each eye) of data: four values of δ and corresponding d's in each set.
    a. Have each student plot δ (inches) vs d (feet) for each eye (the same axis can be used for plotting both sets of data; use color pencils or different symbols to distinguish between left- and right-eye data). Reference is made on a later problem sheet in UNIT III to this graph which, ideally, yields a straight line through the origin.
    b. Have each student compute the Snellen Ratio for each of the eight measurements.
    c. The lesson can be tied together by relating the graph obtained, the Snellen Ratio, S-type Similar Triangles, Clinic Eye Test, etc.

B. Importance of light

3. Sharpness of vision(continued)

d. This description of the normal eye can be represented by the following diagram (not drawn to scale):



THIS IS THE BASIS OF THE SNELLEN RATIO AND THE CLINIC EYE CHART

e. By the use of S-type similar triangles it is evident that the normal eye resolves a δ of 1/32 inch at a d of 10 feet:

$$\frac{1/32 \text{ (inch)}}{1/16 \text{ (inch)}} = \frac{d \text{ (feet)}}{20 \text{ (feet)}}$$

and d = 10 feet

Hence, print having a δ of any size (and a corresponding d) can be used for comparing a test eye to the normal eye.

f. Suppose a test eye can distinguish clearly only those letters with a δ of 1/8 inch at a d of 20 feet.

(1) ASK: at what distance d_n would the normal eye resolve this same δ ?

(2) From the diagram for the normal eye,

$$\frac{1/8 \text{ (inch)}}{1/16 \text{ (inch)}} = \frac{d_n \text{ (feet)}}{20 \text{ (feet)}}, \text{ and } d_n = 40 \text{ feet}$$

(3) The SNELLEN RATIO of this test eye is designated as 20/40; what does it mean?

(4) The Snellen Ratio of a normal eye is given as 20/20.

(5) Use (3) and (4) to formulate a definition of Snellen Ratio.

(6) Incidentally, the familiar clinic eye chart is set up for the test eye to be at a distance of 20 feet.

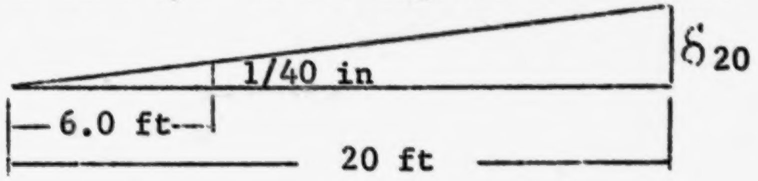
UNIT II. INTRODUCTION TO OPTICS

B. Importance of light

3. Sharpness of vision (continued)

   g. Suppose a test eye resolves letters having a δ of 1/40 inch at a d of 6.0 feet; what is its Snellen Ratio?

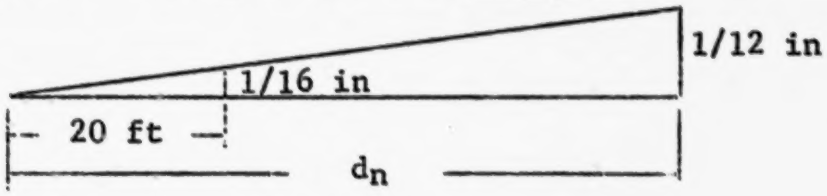
      (1) From the diagram for the test eye, first find δ_{20}, or the smallest spacing that the test eye can distinguish at 20 feet:



$$\frac{\delta_{20}\ \text{(inch)}}{1/40\ \text{(inch)}} = \frac{20\ \text{(feet)}}{6.0\ \text{(feet)}}\ ;\ \delta_{20} = 1/12\ \text{inch}$$

      (2) Then ask: at what distance d_n will the normal eye resolve this same δ ?

      (3) From the diagram for the normal eye,



$$\frac{d_n\ \text{(feet)}}{20\ \text{(feet)}} = \frac{1/12\ \text{(inch)}}{1/16\ \text{(inch)}}\ ;\ d_n = 26.6\ \text{feet}$$

      (4) and the Snellen Ratio of the test eye is 20/26.6.

      (5) In a clinic test, this would show up as 20/30, since no provision is made for any numbers appearing in the Snellen Ratio except integral multiples of ten.

4. Nearpoint of the eye.

   a. See problem sheet on Snellen Ratio for a quick determination.

   b. Nearpoint as it changes with age is subject of the home experiment. **EXPERIMENT ***

   c. Presbyopia and discussion of ciliary muscle.

QUANTITATIVE SCIENCE DATE____________ NAME______________________

1. Define SNELLEN RATIO in a clear, concise statement. Draw the appropriate diagram for an eye with normal vision.

2. a. If a test eye can barely distinguish between strands of wire spaced one-eighth of an inch apart at a distance of 40 feet, what is its Snellen Ratio?

   b. At what distance could a person with 20/90 vision barely distinguish the same strands of wire?

   c. What is the Snellen Ratio of an eye that can barely distinguish between these same strands at a distance of 37 feet?

3. An eye chart revealed a student had a Snellen Ratio of 20/30 in each eye.

   a. Find the limit of resolution between two objects at a distance of 48 feet.

   b. At what distance could this same student resolve two lines spaced 1/64 inch apart?

4. The smallest spacing between parts of a fighter plane's insignia is 22.0 cm.

   a. At what distance from a person with 20/20 vision will the insignia blur?

   b. At what distance from a person with 20/50 vision?

   c. At what distance from a person with 20/14 vision?

5. a. How far apart must a boy and a girl be standing in order that a person with normal vision who is situated 2000 feet away can just barely see that they are separated?

   b. How far apart must they stand for a person 1200 feet away having 20/25 vision to see that they are separated?

6. The NEARPOINT OF THE EYE is the shortest distance of distinct vision, or the closest point to the eye that objects can be brought into focus. How far apart are the smallest divisions on a ruler that can be read by a person with 20/20 vision at a nearpoint of 4.0 inches? Check experimentally.

7. Check your own Snellen Ratio very quickly by determining the distance away from a meter stick or ruler at which you can no longer distinguish between the smallest marked divisions. Do this for each eye separately.

QUANTITATIVE SCIENCE DATE________________ NAME______________________

SHOW ALL WORK BY EACH PROBLEM; ANSWERS NOT CLEARLY SUPPORTED BY WORK ARE WORTHLESS

1. Sam Centauri has 20/50 vision in each eye. At a distance of 30 feet, Sam barely distinguishes between the bricks in a building wall. How thick is the mortar separating the bricks?

1____________________

2. What is the Snellen Ratio of a person who can read only the top line of a clinic eye chart where the smallest spacing between parts of letters is measured to be 0.156 inches?

2____________________

3.. At what altitude would the limed striped markings of a baseball field become indistinguishable to a helicopter passenger with 20/10 vision? STATE CLEARLY ANY ASSUMPTIONS OR APPROXIMATIONS USED IN YOUR CALCULATIONS.

3____________________

QUANTITATIVE SCIENCE **DATE**________________ **NAME**________________________

As part of basic training in the Army, each recruit must test his ability with the M - 1 rifle on a firing range with a target 400 yards away. Assume 20/20 vision and calculate the minimum size of the target's "bull's eye".

____

In the experiment on sharpness of vision a student reads print having a minimum spacing of 3/8 inch from a distance of 150 feet. What is his Snellen Ratio?

____

UNIT II. INTRODUCTION TO OPTICS (continued)

C. Sources of light

1. Luminous bodies.
   - a. objects which of themselves emit light; inherent sources of light.
   - b. examples of natural luminous bodies: sun, stars, lightning, lightning bugs, etc.
   - c. examples of man-made luminous bodies: lamps, matches, candles, torches, arcs, etc.
2. Illuminated bodies.
   - a. objects visible only in presence of inherent sources of light; objects which require the presence of a luminous body to be rendered visible; objects visible only by reflected light.
   - b. everything we see that is not of itself a source of light.
3. "Darkroom test" -- distinguish luminous from illuminated bodies.
4. Property of being luminous depends on the condition as well as the nature of the material:

   cold iron --- illuminated
   hot iron --- luminous

   DEMONSTRATION
5. Incandescence
   - a. Property of solids to emit light when heated to sufficiently high temperatures.
   - b. For appreciable light, temperature must be greater than 800°Celsius.
   - c. Common light bulb is tungsten (wolfram) wire in glass envelope.
   - d. In demonstration, note differences in color and brightness of light as the temperature is increased (by increasing the current)

   DEMONSTRATION
   (in dark room, control current to clear-glass bulb with a Variac)
6. Non-incandescent sources
   - a. fluorescent: mercury vapor impinging on fluorescent coating. DEMONSTRATION
   - b. carbon arc: light originates in space between electrodes. DEMONSTRATION
   - c. sodium vapor: characteristic yellow. DEMONSTRATION
   - d. these differ from incandescent sources, and differ from each other; detailed study later in physics.

D. Intensity of illumination

1. As a white card (or any other object) is moved closer to a given source of light, the "brightness" of the light falling on the card increases; conversely, this "brightness" falls off as the card if moved away from the light source. — DEMONSTRATION

2. A measure of this "brightness" of an illuminated object is referred to as the Intensity of Illumination.

3. The experiment shows quantitatively how the intensity of illumination changes with distance from the source. — EXPERIMENT **

4. The graphical results obtained now will (later) be analyzed and interpreted.

E. Transmission of light

1. The most obvious property of light is that it gets around.

2. Light is capable of covering very large distances; to earth from the moon, sun, stars, galaxies.

3. Light is capable of traveling through empty space.
    a. needs no material medium for its transmission; evidence?
    b. unlike sound: doorbell in vacuum; the very fact that we can see the doorbell after the sound is no longer transmitted suggests light travels through a vacuum. — DEMONSTRATION (only if not seen before!)
    c. historical: AETHER THEORY and Michelson-Morley Experiment to measure the "aether drift" (probably the most famous null experiment of time)

QUANTITATIVE SCIENCE DATE_____________ NAME____________________

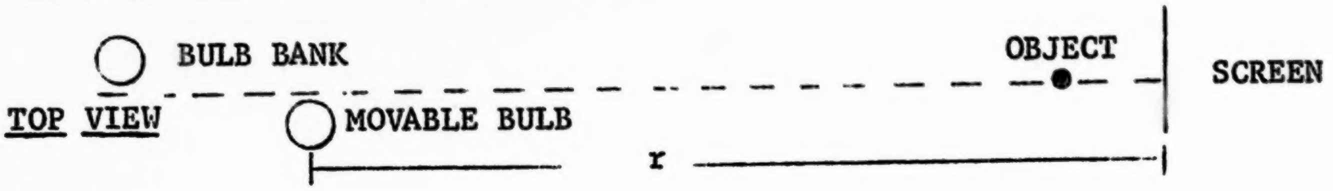
LAB INSTRUCTION SHEET: Illumination

1. If two identical light bulbs (same wattage, matched) are placed slightly off to the side of a central line through a vertical object and perpendicular to a screen, two adjacent shadows of the same "darkness" will be formed.

2. If one of the bulbs is moved in toward the screen, its effect is to brighten the other shadow (and surrounding area) and make its own appear darker. This happens because the illumination produced by a given light source increases as the light source is brought closer, a fact already observed.

3. The object of this experiment is to describe quantitatively the relationship between the brightness and the distance from the light source. This is done by matching the "darkness" of a shadow produced by a different number of light bulbs at a fixed distance to the "darkness" of the shadow produced by one light bulb at different distances. The actual procedure follows below.

4. Place a vertical object in front of a screen at some small, measured, distance. Measure off a distance of _____ meters (your instructor will give you a value for your group) in front of the screen. Place the bulb bank at this position.



5. Select seven matched (how do you determine whether they are matched?) light bulbs of the same wattage. Be sure to record the wattage. Six bulbs are to be placed in the sockets of the vertical bank, and one is to be placed in the socket on the movable stand.

6. Turn on successively one bulb, two bulbs, etc., and find the distance r (cm) from the screen that the movable bulb must be placed to produce a shadow of the same "darkness". Average several readings for each number of light bulbs.

7. Plot a graph of the illumination I (number of bulbs) vs the source-screen distance r (cm).

8. Leave at least two pages in your lab book for (later) analysis of the data.

UNIT II. INTRODUCTION TO OPTICS

E. Transmission of light (continued)

4. Speed of light determinations: four significant measurements.
   - a. Galileo (Italy, about 1620); principal value of the effort was the denial that the speed of light was infinite.
   - b. Roemer (Denmark, about 1675); first successful measurement; ASTRONOMICAL method using the moons of Jupiter.
   - c. Fizeau (France, about 1850); first TERRESTRIAL measurement and basis for most speed of light measurements to follow.
   - d. Michelson (U.S., first American Nobel Prize Winner): ultra-precise measurements spanning the years 1878 - 1926; most famous measurement between Mt. Wilson and Mt. San Antonio, (probably the most precisely measured part of the earth!)

5. Present accepted value for the speed of light in vacuum (the usual symbol for the speed of light is lower case c):
   - a. in metric system, $c = 2.99793 \times 10^8$ meters/second.
   - b. in English units, $c = 1.86282 \times 10^5$ miles/second.
   - c. Usually use the approximations of 3×10^8 meters/sec and 186,000 miles/sec unless greater precision is required (see problem sheet for the error involved)

6. The LIGHT YEAR
   - a. Convenient unit of length for astronomical measurements.
   - b. DISTANCE covered by moving at the speed of light for a time of one year.
   - c. (see problem sheet: one light year is 9.46×10^{15} meters or 5.86×10^{12} miles)
   - d. Alpha Centauri (nearest star) is more than four light years distant from earth; light from this nearest star reaches us at a time more than four years after it was emitted! WE DON'T SEE THE NOW BUT THE THEN OF THE STARS!
   - e. Polaris is more than fifty light years away; the light by which we may observe it TONIGHT left the North Star prior to 1920!
   - f. Arcturus is forty light years away; light from Arcturus was collected, focused on a photocell and used to turn on the lights officially opening the 1933 Century of Progress Exhibition in Chicago - commemorating the 1893 World Fair in the same city.

QUANTITATIVE SCIENCE DATE______________ NAME____________________

SHOW YOUR WORK IN THE SPACE PROVIDED BY EACH PROBLEM.

1. Express one LIGHT YEAR in
   a. meters

1a______________

   b. miles

1b______________

2. Calculate the % difference between the present accepted values of the speed of light and the commonly used values of
   a. 186,000 miles/second

2a______________

   b. 3×10^8 meters/second

2b______________

3. How long does it take light to reach the earth from the sun?

3______________

4. How long did it take light to travel the path in Michelson's experiment to measure the speed of light?

4______________

5. The nearest star, Alpha Centauri, emits light that reaches the earth about 4.3 years later. How far away (in meters) is this nearest star?

5______________

6. How long does it take light to travel the length of this classroom?

6______________

7. Radio waves travel at the speed of light in air or in free space. A radar transmitter, which sends out radio waves of a particular kind, when pointed at the moon receives a reflection 2.7 seconds after the signal is sent. What does this data give as the distance of the moon from the earth?

7______________

UNIT II. INTRODUCTION TO OPTICS (continued)

F. Detectors of light

1. Eye: by definition, the principal detector of light.

2. Photocell.
   a. device in which an electric current is generated when light strikes its surface.
   b. size of current produced depends on the intensity of light striking photocell surface.
   c. common applications: EXPOSURE METER (photography) — DEMONSTRATION
      ELECTRIC EYE (switch) — DEMONSTRATION

3. Photosensitive chemicals.
   a. Substances affected by light; substances in which light produces a chemical change.
   b. Of importance in photography are the SILVER HALIDES:
      silver chloride
      silver bromide
      silver iodide
   c. To observe the sensitivity to light of one of these (silver chloride): — DEMONSTRATION
      (1) prepare the silver chloride by adding a small amount of hydrochloric acid to a solution of silver nitrate.
      (2) the milky white substance formed is silver chloride:

      $$(HCl + AgNO_3 \rightarrow HNO_3 + AgCl)$$

      (3) quickly pour some of the silver-chloride laden liquid into a second container and remove it to a dark place (many cabinets are relatively light tight).
      (4) expose the remainder to bright light (in ordinary room light the reaction is generally slow; a bright lamp brought close or a window admitting sunlight work well).
      (5) note that in the liquid exposed, the white material forms a precipitate and darkens, turns purple-gray in color.
      (6) checking the unexposed liquid reveals a pure white precipitate of silver chloride.
      (7) upon exposing this white precipitate to light, it, too, darkens, turns purple-gray.
      (8) the purple-gray material deposited at the bottom (in both cases) is pure silver --- light reduces the silver chloride salt to metallic silver.
   d. In photographic negatives or prints, the dark areas are deposits of silver resulting from the reaction of a silver halide with light.

F. Detectors of light (continued)

4. Photographic process --- in brief

EXPERIMENT *
(See "Pinhole Image", section H of this unit)

- a. Photographic film, paper, plates, are all coated with a light-sensitive EMULSION in which the silver halide particles are suspended.
- b. EXPOSURE of these particles to controlled light produces a LATENT IMAGE.
- c. The DEVELOPER selectively reduces the exposed silver halide particles to metallic silver (time and temperature are quite critical in controlling developer action, especially with films and plates).
- d. A SHORT STOP or STOP BATH (weak solution of glacial acetic acid) is used to bring the developing process to a sudden, uniform halt.
- e. The FIXER or HYPO (sodium thiosulfate, $Na_2S_2O_3$) dissolves the unexposed silver halide grains but does not affect the silver particles.
- f. A WASH in freely running water removes residual chemicals from the film, plate, or the fibers of photographic paper.

5. Other detectors of light

- a. Phosphorescent materials --- absorb and emit.
- b. Solar cells.
- c. Plants and photosynthesis; change of light conditions for forced blooming.
- d. Artificial "days" and chicken egg production.

QUANTITATIVE SCIENCE DATE__________ NAME____________________

Select by letter the item from the LIST that best fits the numbered STATEMENT

LIST

A. illuminated
B. photocell
C. incandescence
D. Fizeau
E. luminous
F. Michelson
G. medium having mass
H. Morley
I. viscosity
J. 3.00×10^8 meters
K. wolfram
L. Roemer
M. Snellen
N. transmits light
O. 5.86×10^{12} miles
P. density
Q. Einstein
R. inertia
S. fluorescence
T. luminescent

STATEMENT

1. First American Nobel Prize winner 1________
2. Astonomical method for the speed of light 2________
3. Sharpness of vision 3________
4. Assumed essential feature of aether 4________
5. Light year 5________
6. High-temperature property of solids 6________
7. Inherent capability of emitting light 7________
8. Mass per unit volume 8________
9. Tungsten 9________
10. Device in which light generates an electric current 10________

CLEARLY MARK EACH STATEMENT TRUE OR FALSE

11. The moon is an incandescent source of light 11________
12. When the length of the pendulum is increased, the frequency decreases 12________
13. Our current classroom definition of light is a theoretical one 13________
14. Silver nitrate is a halide of importance in photography 14________
15. When the length of a pendulum is decreased by a factor of two, the period is halved 15________

TO RECEIVE CREDIT FOR A PROBLEM YOU MUST SHOW EACH STEP IN YOUR THINKING. CLEARLY SHOW THE USE OF CONVERSION FACTORS

16. If the average red blood cell has a diameter of 7.5 microns, express the order of magnitude of this size in inches.

16__________

17. What kind of a quantity is the product VOLUME X DENSITY?

17__________

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18. If the mass density of mercury is 13.6 gms/cm^3, what is the mass of each fluid ounce (5 fluid ounces is equal to 150 cm^3)?

18__________

19. If Mach 1 is equivalent to a speed of about 750 miles/hour, express Mach 3 in MKS units.

19__________

20. The sun is located at a distance of 1.5 X 10^{11} meters from the earth. Express this distance in light years.

20__________

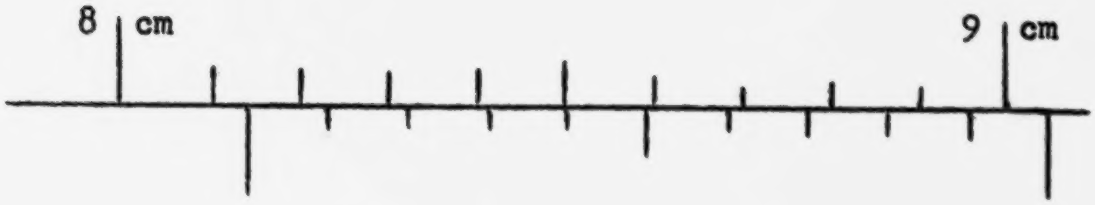
21. The mass density of aluminum in the English system is approximately 5.2 slugs/ft^3. Express this in cgs units.

21__________

22. Side A of a rectangle measures 62.2 ft and side B measures 20.2 meters. Which side is larger and by how much?

22__________

23. The following enlarged diagram represents a length reading on a vernier caliper. What is the measured length?



23__________

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1. Briefly but clearly discuss the origin and meaning of the word SILHOUETTE. Be sure to cite references used.

2. a. Define and discuss the word IMAGE.

   b. We will be working with various optical images. What other kinds are there? Discuss.

   c. Describe completely a PANTOGRAPHIC IMAGE.

3. Make a clear statement of the theorems pertaining to:

   a. S-type similar triangles

   b. P-type similar triangles

   Each statement should be accompanied by a clearly labeled diagram.

UNIT II. INTRODUCTION TO OPTICS (continued)

G. Shadows and the propagation of light

DEMONSTRATION

1. Experimental arrangement for this investigation
   a. SOURCE of light: several different ones to used.
   b. OBJECT: wire mesh (1/16" wire in 1" squares has proved quite satisfactory) or cross bars, in the path of light.
   c. IMAGE of object: shadow produced on a screen; silhouette image.
2. Using a large source (ordinary incandescent bulb, frosted)
   a. Shadow has fuzzy edges; image is very poorly defined.
   b. As source is brought very close to the object, image loses all definition.
3. Using a small source (flashlight bulb mounted on a battery)
   a. Shadow has sharp edges; image is well-defined.
   b. Image remains sharp and clear even when source is brought close to the object.
4. Conclude
   a. The smaller the source of light, the better the silhouette image.
   b. Expect point source of light to produce sharpest image ideally (impossible to attain in practice).
5. Regard small source here as physical point source
   a. Dimensions are "sufficiently small" to approximate an ideal point source.
   b. It is possible to achieve any size physical point source required or desired; for our purposes, the few-millimeter flashlight-bulb source is small enough.
   c. Examples of physical point sources in application: carbon arcs, zirconium crater arcs, etc.

UNIT II. INTRODUCTION TO OPTICS

G. Shadows and the propagation of light (continued)

DEMONSTRATION

6. Using small source, note changes in the image size (and hence position on screen) as the source is moved toward or away from the object (keeping the object-to-screen distance constant)
    a. At various source distances, connect a string through any chosen object point and the corresponding image point; extend the string back to the physical point source.
    b. Observe: POINT SOURCE, OBJECT POINT, AND CORRESPONDING IMAGE POINT ALWAYS LIE ON THE SAME STRAIGHT LINE.
    c. This experiment was performed in air; if this experiment were repeated in an atmosphere of helium, or carbon dioxide (or carbon monoside!), or under water, in alcohol, or in Duco Cement, as long as the light remains in the same medium, the same result as that stated in (b) is obtained.
    d. Hence the conclusion: IN A GIVEN MEDIUM, LIGHT TRAVELS IN STRAIGHT LINES.

7. This basic property of light referred to elegantly as THE RECTILINEAR PROPAGATION OF LIGHT (R.P.L.)

8. Effect of source size and shape
    a. Point source gives sharp image.
    b. Large source gives fuzzy image.
    c. Line source (showcase bulb) gives fuzzy image except in direction of line; cut down "line size" of source by moving two cards toward each other along filament's long dimension. DEMONSTRATE
    d. Use POINT-EXTENDED SOURCE device to demonstrate: EXTENDED SOURCE IS AN AGGREGATE OF POINT SOURCES each producing its own sharp image in a slightly different place (according to R.P.L.); this slight displacement of each sharp shadow is responsible for lack of definition ("fuzziness" of the image. DEMONSTRATE *

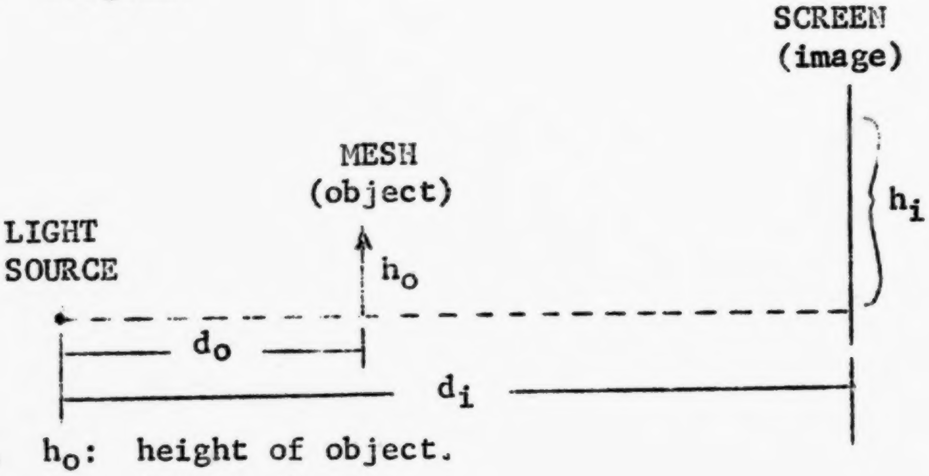
G. Shadows and the propagation of light (continued)

9. Quantitative experiment: Silhouette Image

a. Experimental arrangement: movable screen
wire mesh (or other object)
physical point source

EXPERIMENT *

b. Diagram:



c. h_o: height of object.
h_i: height of image.
d_i: distance of image from source.
d_o: distance of object from source.

d. Measure and record d_o and h_o; these do not change.

e. With d_o fixed, change d_i by moving the screen to several different positions along a line.

f. Measure the height hi of the silhouette image for each value of d_i.

g. Plot h_i vs d_i (h_i along vertical axis,
d_i along horizontal axis).

H. Pinhole Image

1. Observe with either pinhole viewers or demonstration camera (large cardboard box with translucent plastic or glass screen at back):

DEMONSTRATION

a. Image formed is inverted.
b. Image size varies as object position is changed.

2. Quantitative experiment: Pinhole Image

EXPERIMENT *

a. Construct pinhole camera (use photographic enlarging paper rather than film).

b. Measure and record the length of the camera (from pinhole to the photographic paper).

c. Take five pictures of designated object at five different distances; identify each picture by this object distance d_o (in meters).

d. After processing prints, measure the image height h_i (in cm) on each.

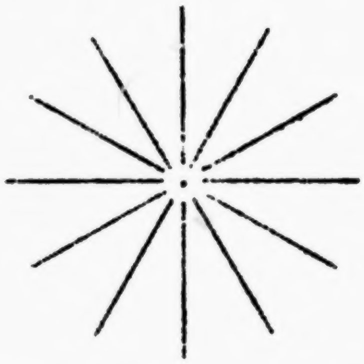
e. Plot a graph of h_i vs d_o (h_i along the vertical axis and d_o along the horizontal axis).

UNIT II. INTRODUCTION TO OPTICS (continued)

I. Parallel light

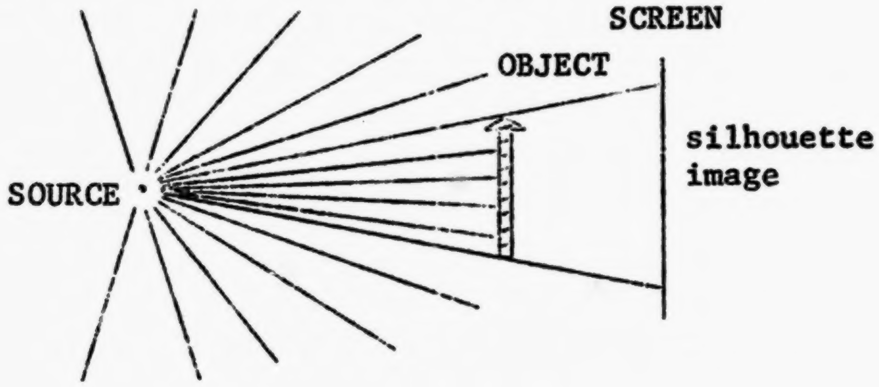
1. Because of the rectilinear propagation of light (a fact already established), the path of light can be represented by straight lines (GEOMETRICAL OPTICS: application of straight-line geometry to the path of light and its consequences)

2. Point source diagram
   - a. Light emitted in all directions.
   - b. Emitted light completely fills three-dimensional space.
   - c. Straight lines represent some of this light.



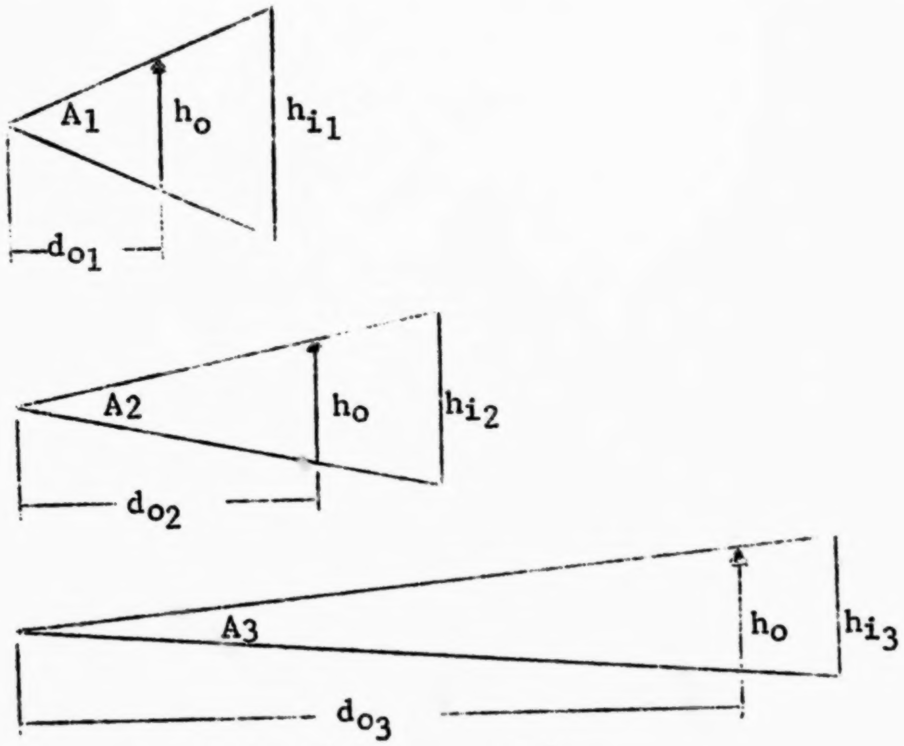
3. If an object is placed in the path of this light,
   - a. Only a portion of the light emitted by the source is intercepted.
   - b. This intercepted light (in one dimension) falls within the triangle formed by the lines of light drawn past the edges of the object.
   - c. Light outside this triangle defines the silhouette image on a screen placed beyond the object.

DEMONSTRATION



I. Parallel light (continued)

4. Observe the change in the angle A and the image height h_i as the object distance d_o from the source is increased:



a. $d_{o_1} < d_{o_2} < d_{o_3} \cdots < d_{o_n}$

b. $A_1 > A_2 > A_3 \cdots > A_n$

c. $h_{i_1} > h_{i_2} > h_{i_3} \cdots > h_{i_n}$

5. As the object is moved farther and farther from the source,

   a. the "lines of light" past the object edges become less and less divergent.

   b. these "lines of light" become more and more nearly parallel.

   c. the image height h_i becomes more nearly equal in size to the object height h_o.

6. In the theoretical limit, that is, when the light source approaches infinity,

   a. the "lines of light" defining the image become parallel and the image size approaches the object size.

   b. symbolically, as $d_o \longrightarrow \infty$, $h_i \longrightarrow h_o$

7. This concept is used to define PARALLEL LIGHT:
   LIGHT FROM A SOURCE OF INFINITY

8. For experimental purposes, a physical infinity is achieved when the source is far enough removed so that there is no measurable difference between h_i and h_o; such light can be considered parallel and certainly can be represented by parallel lines.

J. Beams, pencils, rays

1. Beam
    a. directed light.
    b. light to which some direction can be assigned, with no specification as to how broad or how closely confined the light is to a single direction.
2. Pencil
    a. physical limit of a beam.
    b. answers question: to what extent can a beam be actually confined to a single direction?
    c. difficulty of geometrical spreading; failure of attempts to confine light to same small beam width over a distance of consecutive apertures.

DEMONSTRATION

    d. pencil of light is a physical reality; can be seen and produced in the laboratory.
    (e. Anticipate questions on LASER and beam spreading)
3. Ray
    a. theoretical limit of a pencil of light.
    b. ray has extension in length only; no width or cross section.
    c. OPTICAL EQUIVALENT OF A STRAIGHT LINE SEGMENT.
    d. ray is a mental construct; cannot be produced or observed in the laboratory.
    e. ray is merely a convenient and meaningful way to represent lines of light.
    f. Geometrical Optics --- Ray Optics.

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SELECT BY LETTER THE ITEM FROM THE LIST THAT BEST FITS THE NUMBERED STATEMENT:

LIST

A. Christopher Morley
B. luminous
C. incandescence
D. Armand Fizeau
E. 1100 feet/second
F. Albert Einstein
G. 750 miles/hour
H. Dayton Miller
I. fluorescence
J. Albert Michelson
K. 3 X 10^8 meters/second
L. illuminated
M. Olaf Roemer
N. 186,000 miles

STATEMENT

1. Astronomical method for the speed of light 1________
2. First American Nobel Prize winner 2________
3. Objects visible by reflected light 3________
4. Speed of light 4________
5. High temperature property of solids 5________

CLEARLY MARK EACH STATEMENT TRUE OR FALSE:

6. The moon is an incandescent source of light 6__________
7. The Michelson-Morley Experiment supported the aether theory 7__________
8. Our current classroom definition of light is a theoretical one 8__________
9. Because of the high value of the speed of light, light from distant stars reaches earth within a few minutes after it is emitted 9__________
10. Armand Fizeau made the first successful measurement of the speed of light 10__________

CLEARLY SHOW YOUR WORK IN THE SPACE PROVIDED BY THE PROBLEM; USE DIAGRAMS WHERE THEY MAY BE NECESSARY OR HELPFUL!

11. At what distance can the normal eye distinguish between two marks separated by 15/32 inch?

11__________

12. What is the Snellen Ratio of a test eye that can resolve letters with a minimum spacing of 5/64 inch at 20.0 feet?

12__________

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13. Find the Snellen Ratio of an eye capable of separating 2.78 millimeters from a distance of 10.65 meters.

13____________

14. If an eye tests at 20/16, what is the smallest spacing in letters it can distinguish on a clinic eye chart 20.0 feet away?

14____________

15. A student measures a $\delta = 0.02$ inches for letters he can read at a distance of 4.0 feet. Find his Snellen Ratio.

15____________

16. The sun is located at a distance of 1.5×10^{11} meters from the earth. Express this distance in light years.

16____________

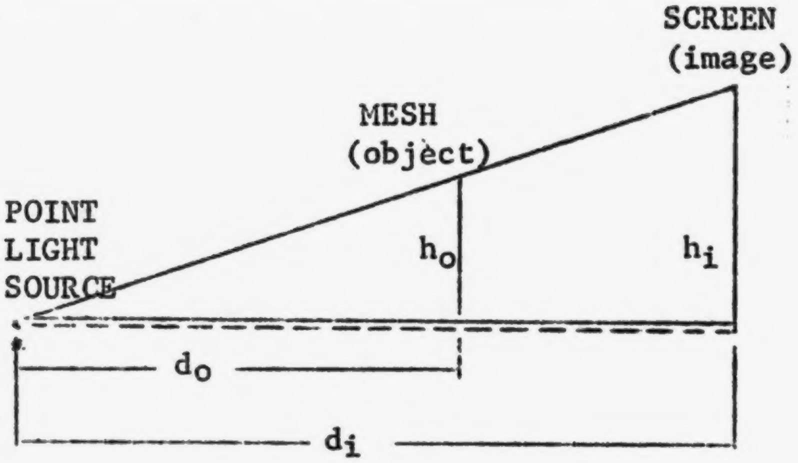
SILHOUETTE AND PINHOLE IMAGE ANALYSIS

The two sections that follow in the outline (K. Silhouette Image Analysis, and L. Pinhole Image Analysis) are included here for continuity. However, both serve as excellent applications of the graphical analysis that follows in the next unit.

It is, therefore, suggested that this final treatment of the Silhouette and Pinhole Image be postponed until DIRECT and INVERSE VARIATION, respectively, are concluded in Unit III. Since the analyses pertain directly to the laboratory results, it would be wise to instruct the students when the experiments are performed to leave several pages blank in the laboratory notebook for the later completion of this material.

K. Silhouette image: analysis of experimental results.

1. Diagram and quantities involved:



d_o constant
h_o constant
d_i independent variable
h_i dependent variable

2. Property of light previously established: point source, object point and corresponding image point lie on the same straight line.

3. So. draw rays of light passing upper and lower edges of object.

4. Then h_o and h_i are parallel lines; rays drawn cut h_o and h_i and intersect, satisfying the conditions for S-type similar triangles, (S for silhouette). Hence, corresponding sides of triangles formed are proportional,

5. giving $h_i/h_o = d_i/d_o$; multiplying both sides by h_o : $h_i = (d_i/d_o)h_o$

$$\text{and } h_i = (h_o/d_o)d_i$$

6. Since both quantities h_o and d_o are constant, write $h_i = k\ d_i$, where k is h_o/d_o , a constant.

7. Compare to experimental results: graph if h_i vs d_i is straight line through the origin.

8. Form is $y = mx$, where m is given by $\Delta y/\Delta x$, the slope of the line.

9. Hence the slope of the h_i vs d_i straight line gives the constant h_o/d_o;

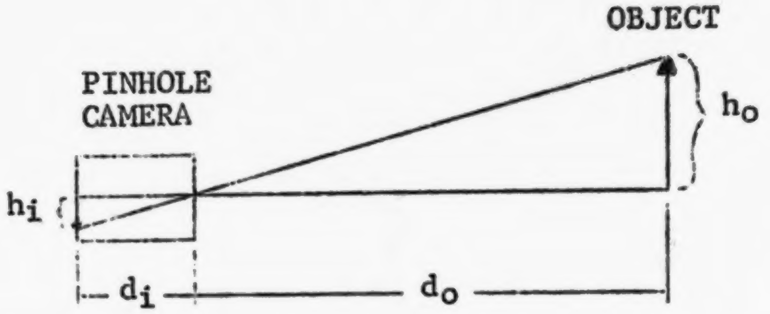
$$\Delta h_i/\Delta d_i = h_o/d_o$$

10. Refer to graph and data; compute the slope, insert values and check.

L. Pinhole Image: analysis of experimental results.

1. Diagram and quantities involved

h_o constant
d_i constant
d_o independent variable
h_i dependent variable



2. Since light travels in straight lines, draw ray of light from top and bottom of object, respectively; each must pass through small pinhole (essentially a point) to reach photographic paper and form pinhole image.

3. Note that h_i and h_o are parallel lines; rays drawn cut these parallel lines and themselves intersect between the lines; therefore P-type similar triangles formed (P for pinhole, get it?)

4. Hence, since corresponding sides are proportional, $h_i/h_o = d_i/d_o$,

$$\text{and } h_i = d_i h_o(1/d_o)$$

5. Since both d_i and h_o are constant, write $h_i = K\ (1/d_o)$, where $K = d_i h_o$

6. This expression for h_i as a function of d_o is of the form $y = k(1/x)$; hence, h_i varies inversely with d_o, and a hyperbola results for the graph of h_i vs d_o.

7. Analyze the experimental data; is the graph of h_i vs d_o recognized as a hyperbola?

8. Write the expression relating h_i and d_o, complete with units.

9. Compare the results predicted by the rectlinear propagation of light (b through f above) and the actual experimental data.

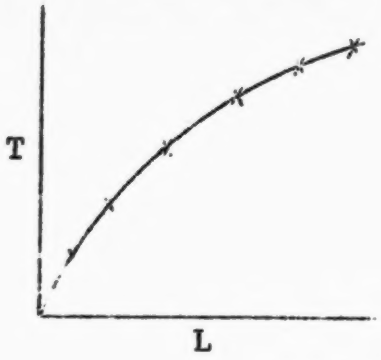
10. What value does the data predict for the height of the object, h_o? Compare to the measured object height supplied by the instructor.

UNIT III. FUNCTIONS, RELATIONS, VARIATION, AND GRAPHS

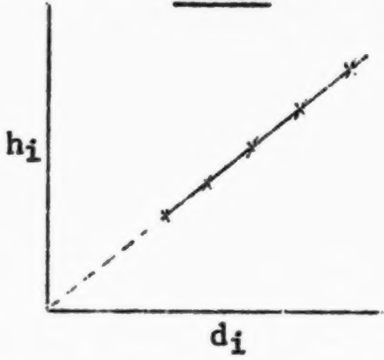
A. Introduction

1. Recall graphs of data from some laboratory experiments already performed:

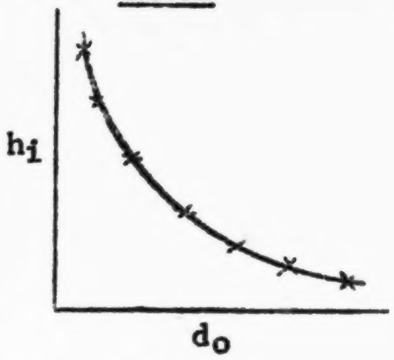
PENDULUM



SILHOUETTE IMAGE



PINHOLE IMAGE



   a. In each, a change of one quantity (length L, image distance d_i, and object distance d_o, respectively) introduces a change in a second quantity (period T, silhouette image height h_i, and pinhole image height h_i, respectively)

   b. In each, the two quantities are related.

2. The aim here --- the purpose of this unit --- is to analyze graphs of data; that is, examine graphical data and attempt to write a mathematical statement expressing the relationship between the quantities.

   a. In a sense, science may be defined as a "search for regularity".

   b. Giving expression to this regularity or observed patterns in nature is what is meant by scientific laws.

   c. Frequently --- and fortunately --- these laws about the behavior of some piece of the universe can be expressed concisely by a simple mathematical sentence.

   d. The present unit concerns itself with going from laboratory observations to the mathematical expression of these findings by way of graphs.

3. Graphical analysis is directly in keeping with the quantitative aspect of this course.

4. Lord Kelvin (about 1890): "... when you cannot express (what you know) in numbers (quantitatively), your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge but you have scarcely ... advanced to the stage of Science..."

UNIT III. FUNCTIONS, RELATIONS, VARIATION, AND GRAPHS (continued)

B. Numbers, variables, and coordinates

1. In math, we speak of an ORDERED NUMBER PAIR, like (3,5), where 3 and 5 are just numbers which can be marked off along scales (number lines or axes).

2. Because we are primarily concerned with measured quantities, here the elements of an ordered pair are usually PHYSICAL QUANTITIES, like (3 seconds, 5 meters), where the first element is an interval of time and the second a distance measure; each element of the ordered pair is not merely a number, but a number plus associated units as well.

3. The symbols used in discussing these elements of the ordered pair are called variables.
    a. In math, most often lower case letters are used as variables; usually, (x, y), in that order.
    b. Here, symbols are commonly chosen to designate the physical quantities measured: (d_o, h_i) or (object distance, image height) in the case of the pinhole image.
    c. Variables are needed and used to state general truths which express relationships between the elements of ordered pairs in a given set:

        MATH EXAMPLE: Given the set of ordered numbers pairs, (0,3), (1,4), 2,5), (3,6), ...
        The general truth which relates the number pairs of the set is expressed by using the variables (x,y) and is "y is equal to x plus 3".

        LAB EXAMPLE: Given values of the variables (d_i, h_i) in the Silhouette Image Experiment, (24 cm, 3.0 cm), (32 cm, 4.0 cm), (40 cm, 5.0 cm), (48 cm, 6.0 cm), ...
        The general truth from the set of ordered pairs: "h_i is is equal to one-eighth d_i".

4. Variables are also referred to as coordinates (particularly for use in graphing).
    a. x-coordinate (or first coordinate).
    b. y-coordinate (or second coordinate).

5. Variable derives its name from the fact that it may take on different values: VARIABLES VARY!
    a. For each different number assigned to x, y may take on a different value.
    b. Distinguish between variables and constants: a constant is a quantity whose value does not change (in a given problem).
    c. Distinguish variables and constants in examples above (3c).

C. Function

1. General notion

a. Statement of the dependence of one quantity on another (or others).
EXAMPLES:
The PRICE of watermelons depends on the SUPPLY.
The LENGTH of a flagpole's shadow depends on the TIME of day.
My GRADE in this course depends on my ABILITY and EFFORT.
The NUMBER of days in detention depends on the SERIOUSNESS of the misconduct.

b. Relation between the values of one quantity in terms of the values of another (still not precise, but closer to the mathematically defined idea of a function).
EXAMPLES:
The AREA of a square depends on the LENGTH of its side.
The PERIOD of a pendulum depends on its LENGTH.
The HEIGHT of the pinhole image depends on the OBJECT DISTANCE.

2. Defintion of a function

a. A quantity y is a function of a quantity x if the values of y depend uniquely on the values of x in a prescribed manner. (It may be advisable to point out that the same statement with the word "uniquely" deleted defines a relation).

b. For a given quantity x, there is one and only one second quantity y which stands in the given relation to x.

3. y is a function of x means that the values of y depend on the values of x according to a stated rule.

a. x is called the INDEPENDENT VARIABLE, or the quantity to which values are assigned.

b. y is called the DEPENDENT VARIABLE, or the quantity whose values are determined by the values assigned to the independent variable.

c. Here, we choose values for one quantity (the independent variable), then measure the corresponding values of the other quantity (the dependent variable).

EXAMPLES:
pendulum: choose values for L, measure resulting values of T
pinhole: choose values for d_o, measure resulting values for h_i
silhouette: choose values for d_i, measure resulting values for h_i

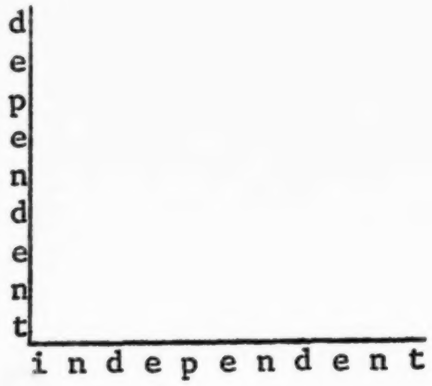
UNIT III. FUNCTIONS, RELATIONS, VARIATION, AND GRAPHS

C. Functions (continued)

4. Functions may be represented by TABLES, GRAPHS, EQUATIONS

   a. Data tables: usual way to record and organize the quantities measured in an experiment (actually gives a set of ordered pairs).

   b. Graphs: by convention, the independent variable is assigned the horizontal axis (x-axis), and the dependent variable the vertical axis (y-axis).



   c. Equations: functional notation, $y = f(x)$, is the symbol used to represent the value of the function corresponding to a particular value of x.

EXAMPLES:

Given the function, $y = x + 3$; $f(x) = x + 3$:
$f(0)$ represents the function at a value of x equal to 0,
so $f(0) = 3$;
$f(2)$ is a directive to substitute 2 wherever x appears
in the expression $f(x) = x + 3$, so $f(2) = 5$.

Suppose we have the function $y = 2.5\ x^2$; $f(x) = 2.5\ x^2$;
Then $f(0) = 0$
$f(1) = 2.5$
$f(2) = 10.0$
$f(5) = 62.5$

5. Note that mathematics has extended the idea of a function far beyond what will be studied here; here the treatment is limited to just that which is sufficient for our needs in analyzing graphical data.

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1. One whole number y is related to another whole number x so that y is three less than twice x.

   a. State the relation as an equation.

   b. State the relation as a table for all positive x values from 0 to 10.

   c. State the relation as a set of ordered pairs for values of x from -5 to +5.

   d. Evaluate f(12).

2. A function is given by the following set of ordered pairs:

   $\left\{(0,0),\ (1,5),\ (2,10)\ (3,15)\right\}$

   Express this function as an equation.

3. If $y = 2x - 1$

   a. Find f(0).

   b. Find f(-3).

   c. Find f(5).

4. If $3x - 5y = 2$

   a Express y as a function of x.

   b. Evaluate: f(2), f(-10), f(0).

   c. For what value of x does the function have a value of 8/5?

5. A function that is important in both science and math can be expressed (approximately) by the equation

   $y = 2.72^x$ where x is an exponent, not necessarily an integer

   a. Plot the function for positive values of x from 0 to 3 (integers only).

   b. From the graph, obtain a value for f(0.75).

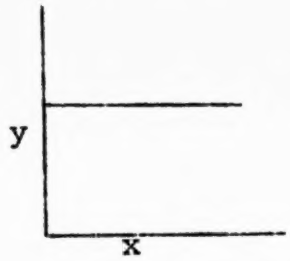
   c. From the graph, obtain a value for f(1.5).

   d. For a value of y equal to 20, find ? of f(?).

UNIT III. FUNCTIONS, RELATIONS, VARIATION, AND GRAPHS (continued)

D. Graphs of data: determination of the functional relation between the variables

1. If the graph of y vs X (note the order: dependent variable vs independent variable) gives a HORIZONTAL STRAIGHT LINE, or a straight line parallel to the x - axis,

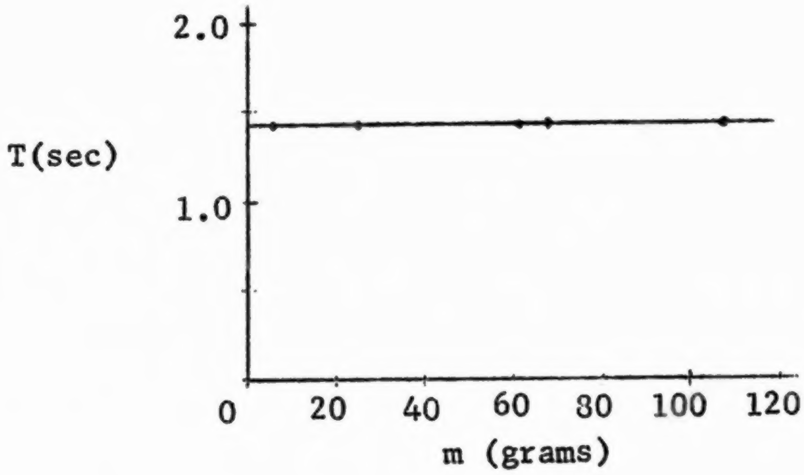


   a. y remains at the same value for any x; y values do not change when x takes on different values.

   b. y is said to be a constant function.

   c. The equation describing such a line is simply y = k, where k is a constant, a fixed, single value of y.

   d. An example of this kind of result is the graph of the data for the period T of the pendulum as a function of the bob mass m:



The equation of the resulting straight line parallel to the m axis is of the form y = k: T (sec) = 1.41 (sec).

UNIT III. FUNCTIONS, RELATIONS, VARIATION, AND GRAPHS

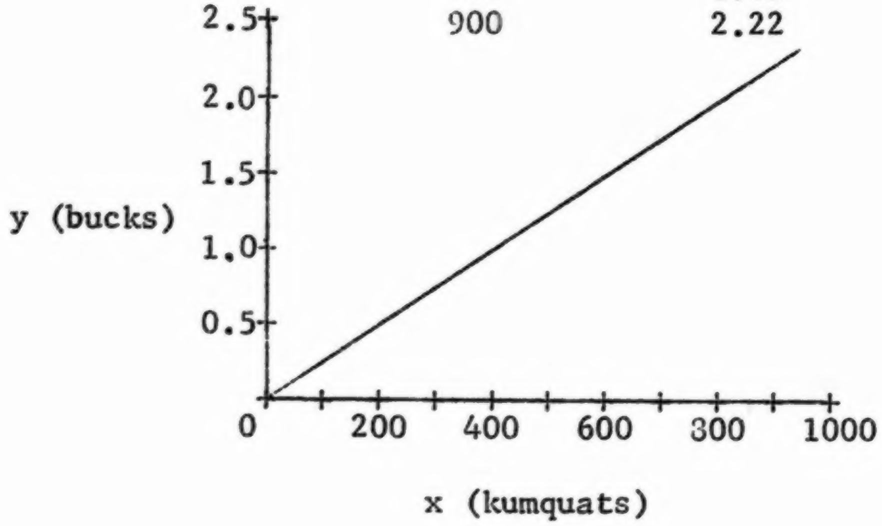
D. Graphs of data (continued)

2. If the graph of y vs x gives a STRAIGHT LINE (not horizontal) THROUGH THE ORIGIN:

EXPERIMENT *
(Big Circles,
C vs r)

a. EXAMPLE: An enterprising young Floridian discovered that the amount of money y (bucks) he earned was a function of the number x (kumquats) he picked; plot the following data:

x (kumquats)	y (bucks)
0	0
150	0.37
300	0.74
450	1.11
600	1.48
750	1.85
900	2.22



b. Because the graph of y vs x gives a straight line, y and x are said to be LINEARLY RELATED; y is a linear function of x.

c. The ratio of the two variables evaluated at any point along the line is a constant: $y_1/x_1 = y_2/x_2 = y_3/x_3 =$ constant.

d. Since this is true for any point on the line, y/x = constant or $y/x = k$, where k is a constant, describes the line.

e. Multiplying both sides of this last expression by x, the equation describing a straight line through the origin can be written $y = kx$.

f. $y = kx$ is the equation of DIRECT VARIATION; y varies directly with x (k in the equation is called the constant of variation).

g. The relationship $y = kx$ is also referred to as DIRECT PROPORTION; y is proportional to x (and k is called the constant of proportionality).

Experiment: BIG CIRCLES AND LITTLE CIRCLES (Part 1)

The essential idea of the BIG CIRCLE part is the measurement of the circumference of circles having different radii. The idea is introduced in discussion:

(1) Students have probably never measured a curved length and almost certainly have never tried to relate the measured circumference to the measured radius (or diameter), but accepted the "well-known relation" on faith.

(2) How can the circumference of large (radius of few meters) circles be measured? (Students have suggested a multitude of methods, many involving either large lengths of flexible wire or great expenditures of time and patience; invariably - - - and usually quickly - - - the suggestion of a calibrated wheel is made).

(3) If P.S.S.C. Hand Strobes are available these can be used as ready-made measuring wheels. If not, a stiff cardboard circle about the size of a dinner plate (or smaller) with a short handle thumbtacked at the center serves the purpose well.

(4) A reminder that the measuring wheel must be calibrated is in order if this is not explicitly brought out in the discussion (this, of course, is readily accomplished by running the wheel n turns on a meter stick). A suggestion: write the words "fractional revolution" on the board during calibration time; this has often spawned the idea of a decimally divided wheel.

Formal instructions for the experiment may be limited to the following:

(A) Measure the circumference C (in meters) as a function of the radius r (in meters) for six circles whose radii lie in the range from 0 - 3.0 meters.

(B) Plot C vs r.

(C) Examine the graph: analyze and conclude.

Make available lengths of string slightly greater than 3 meters. The usually unavailable gym is ideal for this experiment but school corridors serve adequately, although the larger radii permit only semi-circle measurements.

Did you ever think anybody could get excited over "discovering" π?

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1. Examine your graph of data (h_i vs d_i) obtained in the experiment on the silhouette image.

   a. Considering the accuracy involved in performing the experiment, do you think drawing a straight line to represent the data points is justified? In any case, draw the best fitting straight line through the data points so that its extension passes through the origin.

   b. For each point, compute the ratio h_i/d_i.

   c. Are the values of the ratio constant? Compute the largest percent difference.

   d. Compute the slope of the line over the values of measured points.

   e. Compare the slope values to the results obtained in (b) above.

   f. Write the expression relating h_i and d_i (h_i as a function of d_i). Be sure to specify units.

   g. Can you advance any ideas about the constant appearing in the expression?

2. Examine your graph of cylinder heights measured in centimeters and in inches in the very first experiment on length.

   a. Can the data be represented by a straight line through the origin?

   b. For each data point, compute the ratio of centimeters to inches.

   c. Compute the slope of the line over the values of measured points.

   d. Write the expression relating centimeters and inches.

3. Examine your graph of δ vs d in the Snellen Ratio experiment.

   a. Can the data be represented by a straight line through the origin?

   b. For each point compute the ratio of delta to d.

   c. Compute the slope of the line over the values of measured points.

   d. Write the expression relating δ and d.

   e. Can you arrive at the same result geometrically?

4. Draw a graph of feet/second vs miles/hour, choosing values for miles/hour in intervals of 10, up to a maximum of 100 miles/hour.

   a. Compute the slope of the line.

   b. Write feet/second as a function of miles/hour.

D. Graphs of data

2. If the graph of y vs x gives a STRAIGHT LINE THROUGH THE ORIGIN (continued)

h. SLOPE OF A LINE

(1) let (x_1, y_1) and (x_2, y_2) be coordinates of any two points along a line.

(2) the SLOPE m of the line is defined as a change in y divided by a corresponding change in x, or

(3) $$\text{slope } m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

(4) this can be written using the Greek symbol Δ (delta):

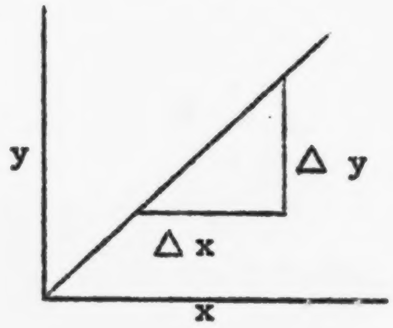
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \Delta y/\Delta x$$

(5) these differences (or deltas) in the values of the variables are called increments.

(6) note that the increments must be corresponding changes in y and x values.

i. Graphical computation of the slope: m is given by the ratio of the vertical side Δy to the horizontal side Δx of a geometrical triangle.

(compute values of the slope on example graph)



j. Possible values of the slope: positive, negative, zero, undefined (show graphical meaning).

k. Physical significance of the slope: SLOPE FROM A GRAPH OF DATA HAS UNITS AND REPRESENTS A PHYSICAL QUANTITY. (in the example graph, the units of the slope are bucks/kumquats).

l. Since the slope can be computed using any pair of coordinates on the line, FOR A LINE PASSING THROUGH THE ORIGIN,

(1) let (x_2, y_2) be (x, y), and (x_1, y_1) be $(0,0)$, that is, measure the slope between the origin and any other point on the line

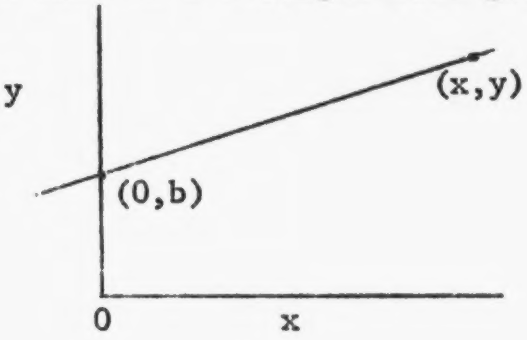
(2) then the slope $m = \frac{y - 0}{x - 0}$, or $m = y/x$

(3) multiplying both sides by x, $y = mx$

(4) Hence, the equation of a STRAIGHT LINE THROUGH THE ORIGIN is $y = mx$ (an extremely useful thing to know and understand!)

D. Graphs of data (continued)

3. If the graph of y vs x gives a STRAIGHT LINE NOT THROUGH THE ORIGIN (general case of a straight-line graph):



a. If the line does not pass through the origin, it crosses the y-axis at some point (0,b).

b. b is called the y-intercept, or the value of y for which x = 0.

c. b can be either positive or negative (above or below the x-axis); b is zero if the line goes through the origin.

d. Let (x,y) be ANY other point along the line.

e. Then, since the slope is defined as $\Delta y/\Delta x$, the slope

$m = \frac{y - b}{x - 0}$; multiplying both sides by x, $mx = y - b$.

f. Writing y as a function of x, $y = mx + b$;
This is the GENERAL EQUATION OF A STRAIGHT LINE
(Understand and remember with all your heart!)

g. $y = mx + b$ is referred to as the SLOPE-INTERCEPT FORM OF THE STRAIGHT LINE EQUATION.

h. Recall that for a straight line through the origin, $y = mx$; this is a special case of $y = mx + b$, where $b = 0$.

i. APPLICATION:

(1) every time a graph (data or otherwise) is a straight line, the relation between y and x can be immediately expressed as $y = mx + b$, where the values of m and b are obtained directly from the graph;

(2) whenever the form of the relation between y and x is $y = mx + b$, a plot of y vs x will give a straight line.

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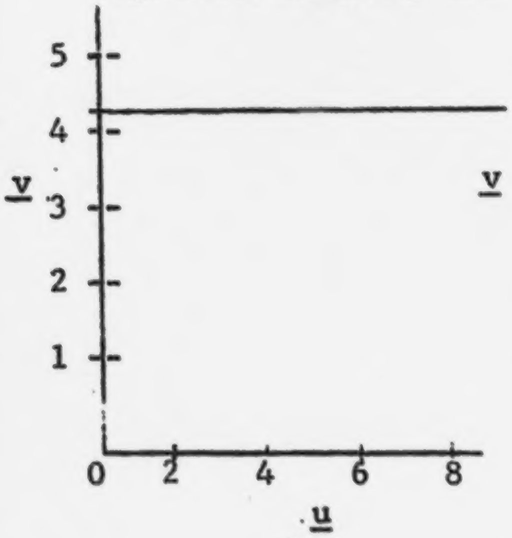
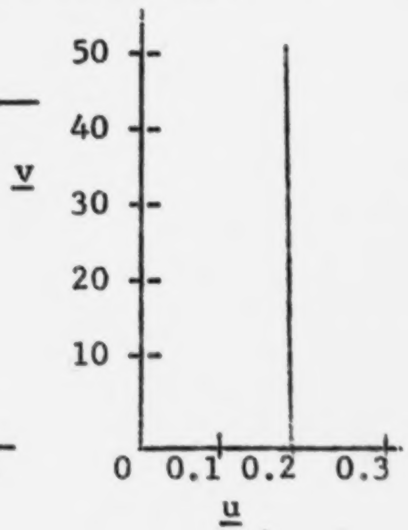
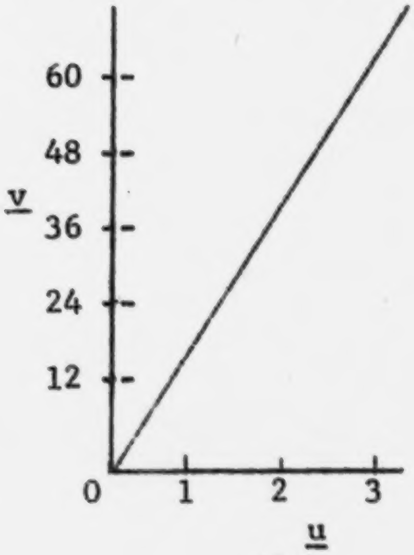
1. In each case below, investigate the slope of the line to write an appropriate equation relating the variables:



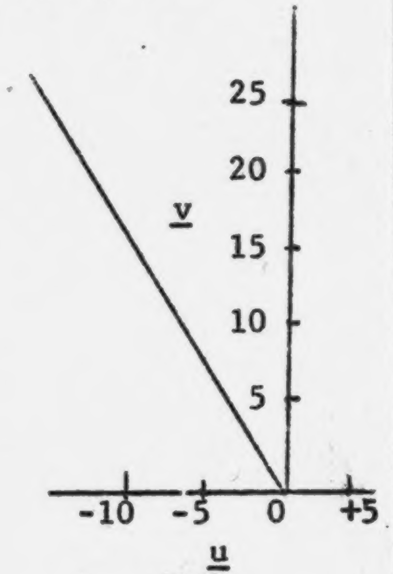
(1a)



(1b)



(1c)



(1d)

2. In each case below, express y as a function of x; if the function is linear, find the slope and the y-intercept.

   a. $4x - 3y = 12$.

   b. $6 + 2x = y - 9$.

   c. $1.6y - 0.88x = 0.12x - 0.30y$.

3. Given the equations below, using only the given information about the slope and the y-intercept, make a graph of y vs x for each case.

$$\left.\begin{array}{ll} \text{a.} & y = 0.3x \\ \text{b.} & y = 0.3x + 0.5 \\ \text{c.} & y = 0.3x + 1.0 \end{array}\right\} \text{ on same graph}$$

$$\left.\begin{array}{ll} \text{d.} & y = 3x - 1 \\ \text{e.} & y = 3x + 1 \end{array}\right\} \text{ on same graph}$$

f. $y = 5 - 2x$

4. It is a familiar fact that the pressure of the atmosphere changes as the altitude above sea level is increased. The following data shows this variation for altitudes up to 5000 feet (the altitude is given in feet above sea level; the pressure is expressed as inches of mercury).

ALTITUDE (feet)	PRESSURE (inches of Hg)
sea level	29.92
1000	28.86
2000	27.82
3000	26.81
4000	25.84
5000	24.89

a. Plot pressure vs altitude.
b. Find the slope of the line.
c. Find the y-intercept.
d. Write the functional relation between the pressure and the altitude; be sure to specify the units of measurement.

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IN EACH OF THE FOLLOWING EQUATIONS EXPRESS y AS A FUNCTION OF x: IF y AND x ARE LINEARLY RELATED, FIND THE SLOPE AND y-INTERCEPT.

1. $2y = 3x - 4$

1______________________

____

____

2. $3x + 2y = 2$

2______________________

____

____

3. $y - 1.5x = 1.5x - 2y$

3______________________

____

____

4. $2y + 4 = 2x^2 + 4$

4______________________

____

____

5. $5 - 2x = 25 - 5y$

5______________________

____

____

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MATCH EACH NUMBERED STATEMENT WITH THE LETTER OF THE ITEM IT BEST DESCRIBES.

ITEM

A. $\Delta x/\Delta y$
B. (0,b)
C. hypo
D. divergent
E. parallel
F. acetic acid
G. silhouette
H. constant
I. variable
J. short stop
K. ray
L. pencil
M. beam
N. extended source
O. b
P. increments
Q. $\Delta y/\Delta x$
R. $x = 0$
S. $y = 0$
T. $x =$ infinity
U. developer
V. $y = (3/7)\,x - 9$
W. point source
X. direct variation

1. Image produced by interposing an object between a light source and a screen. 1__________
2. Produces sharpest image 2__________
3. Equation of the x-axis 3__________
4. Reduces exposed silver halides to Ag 4__________
5. Optical equivalent of a straight line segment 5__________
6. Light from a source at infinity 6__________
7. Physical limit of a light beam 7__________
8. Quantity whose value does not change 8__________
9. Slope 9__________
10. $y = kx$ 10__________
11. y is a linear function of x 11__________
12. Value of the function for which $x = 0$
13. Small differences in values of the variables 13__________
14. Dissolves unexposed silver bromide particles 14__________
15. Coordinates of the y-intercept 15__________

IN EACH OF THE FOLLOWING EQUATIONS EXPRESS y AS A FUNCTION OF x; IF y AND x ARE LINEARLY RELATED, FIND THE SLOPE AND THE y-INTERCEPT

16. $5 - 2x = 25 - 5y$ 16__________

____

____

17. $2y + 4 = 2x^2 + 4$ 17__________

____

____

18. $y - 1.5x = 1.5x - 2y$ 18__________

____

____

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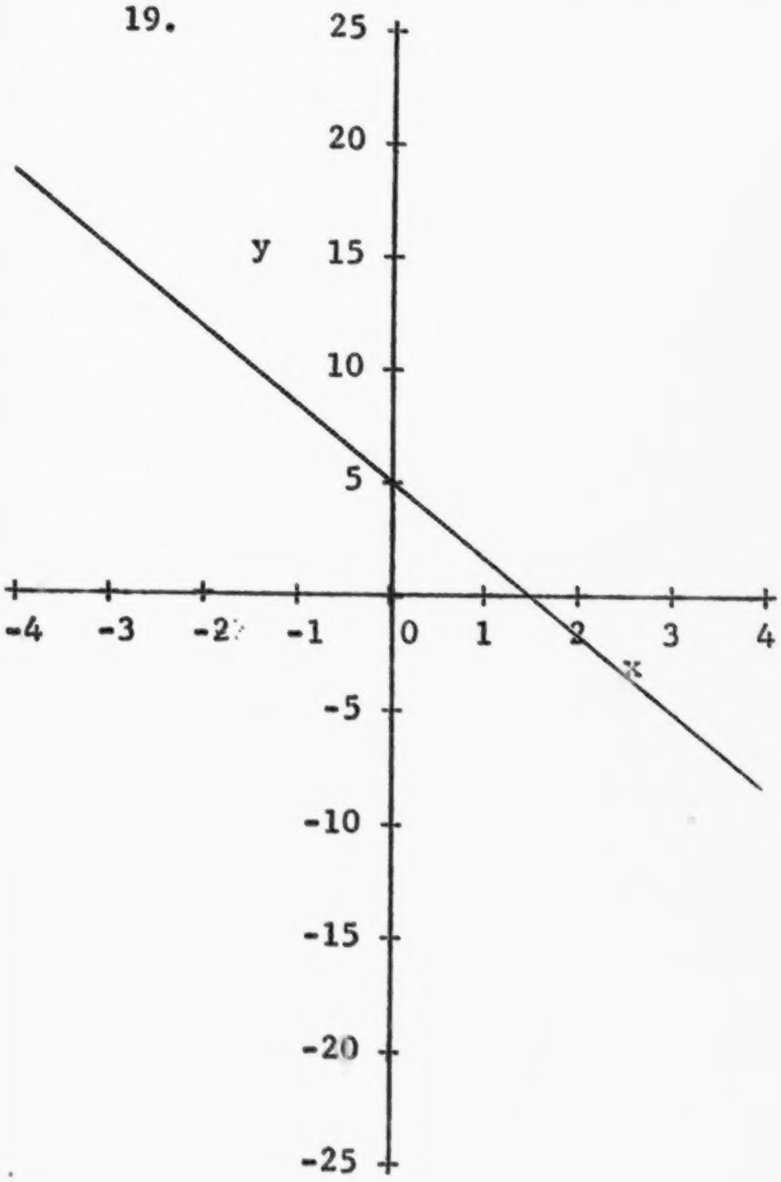
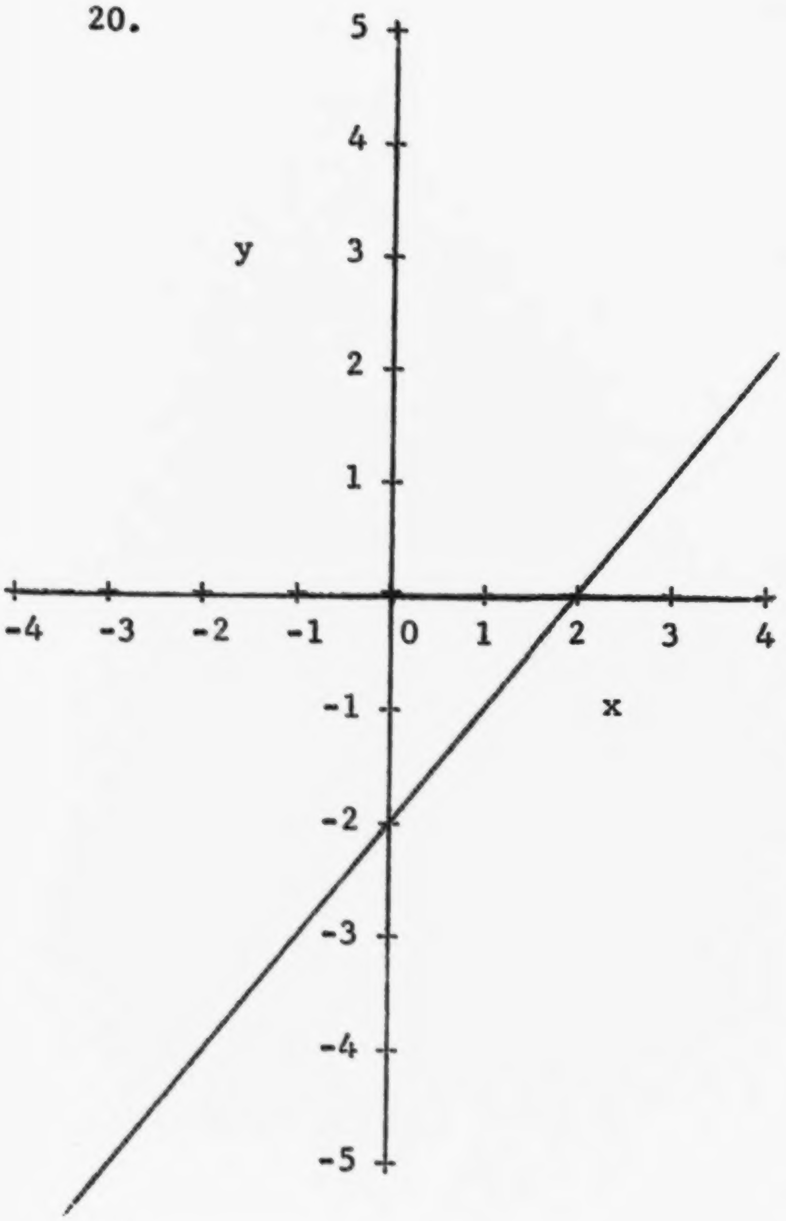
GIVEN THE FOLLOWING STRAIGHT LINE GRAPHS, FROM THE SLOPE AND THE y-INTERCEPT WRITE THE EXPRESSION RELATING y AND x.

19.



20.



19________________________

20________________________

GIVEN THE FUNCTION $y = \frac{1}{2}x + 2\frac{1}{2}$, USING ONLY THE GIVEN INFORMATION ABOUT THE SLOPE AND THE y-INTERCEPT, CONSTRUCT A GRAPH OF y vs x.

21. (to be done on graph paper)

THE FOLLOWING DATA WAS OBTAINED IN AN EXPERIMENT PERFORMED TO FIND THE RELATIONSHIP BETWEEN TWO TEMPERATURE SCALES, CELSIUS AND FAHRENHEIT. PLOT y (°C) vs x (°F). FROM THE GRAPH, WRITE THE FUNCTIONAL RELATION BETWEEN y AND x.

22. (to be done on graph paper) ____________________________________

x (F)	y (C)
-13	-25
32	0
50	10
68	20
140	60

D. Graphs of data (continued)

4. If the graph of y vs x gives a CURVE (not a straight line),

   a. Of the many possible shapes of graphs, a few recognizable forms will be investigated here.

   b. The four special kinds of curves treated here are among the most commonly encountered, the most useful, and the most easily treated at this level of mathematics.

   c. This "RECOGNITION METHOD" requires a reasoned guess, one backed by the logic of relating graphical shapes to mathematical equations ("Chance favors the prepared mind").

   d. A mathematical verification follows by reference to the known analysis of a straight-line graph.

   e. The treatment here of the four special cases easily leads to the analysis of a great number of additional curves.

   f. By GRAPHICAL ANALYSIS here is meant:

      (1) starting with data (experimentally measured quantities);

      (2) plot the variables (graph);

      (3) use the graph to derive the mathematical relation or function between the variables;

      (4) write the final expression relating the measured quantities, COMPLETE WITH UNITS.

5. First special case of a CURVE:

   a. EXAMPLE: data from an experiment in sound (The basis for this data is the fact that the frequency ((physical equivalent of pitch)) depends on the length of a vibrating system. In the case of an organ pipe, it is a vibrating air column that produces the sound. The laboratory organ pipe has a movable piston which allows the length of the column to be varied. Length values were chosen to correspond to notes on the DIATONIC SCALE).

DEMONSTRATION

UNIT III. FUNCTIONS, RELATIONS, VARIATION, AND GRAPHS

D. Graphs of data

5. First special case of a CURVE

a. EXAMPLE: Data from an experiment in sound (continued)

NOTE	LENGTH L (cm)	FREQUENCY f (cycles/sec)
C'	15.6	528
B	16.6	495
A	18.7	440
G	20.8	396
F	23.4	352
E	24.9	330
D	27.7	297
C	31.2	264
B_1	33.2	248
A_1	37.4	220
G_1	41.6	198
F_1	46.8	176
E_1	50.0	165
D_1	55.6	148
C_1	62.4	132

DEMONSTRATION

Plot f (dependent variable) as a function of L (independent variable).

b. Resulting graph is nonlinear, does not pass through the origin, and does not cross any axis (asymptotically approaches both axes).

c. This particular kind of curve is called a HYPERBOLA.

d. Note from the data and from the graph, as the length is increased, the frequency decreases.

e. The relationship between the two quantities here is an example of INVERSE VARIATION or INVERSE PROPORTION; this is expressed generally: y varies inversely with x, or y is inversely proportional to x.

f. Note that the product of the variables is the same for any point along the curve:

$$L_1f_1 = L_2f_2 = L_3f_3 = \ldots = 8.24 \times 10^3 \text{ (cm/sec)}$$

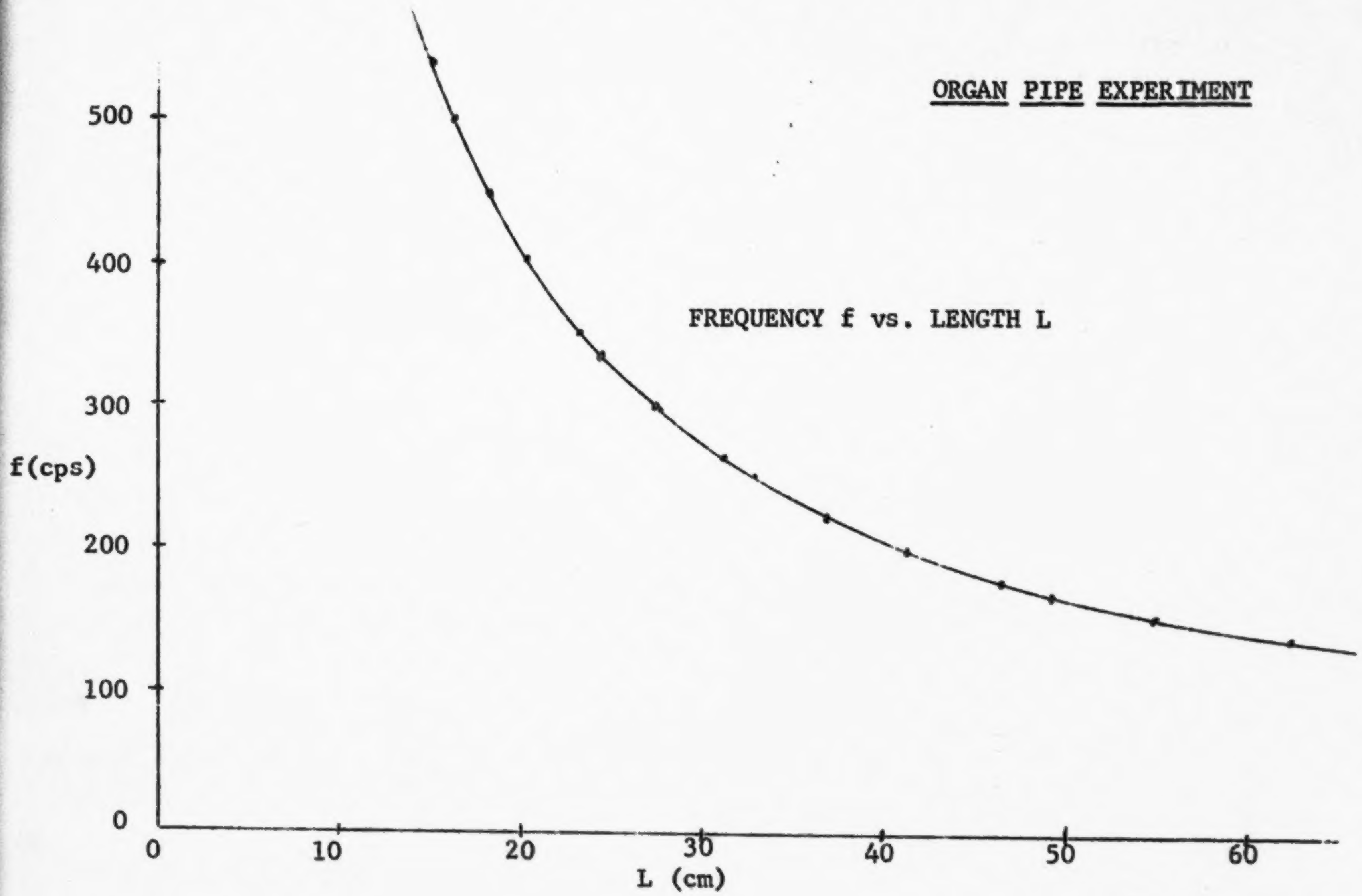
g. The product of the variables is a constant; this is expressed in general for this kind of curve, $xy = k$, where k is a constant.

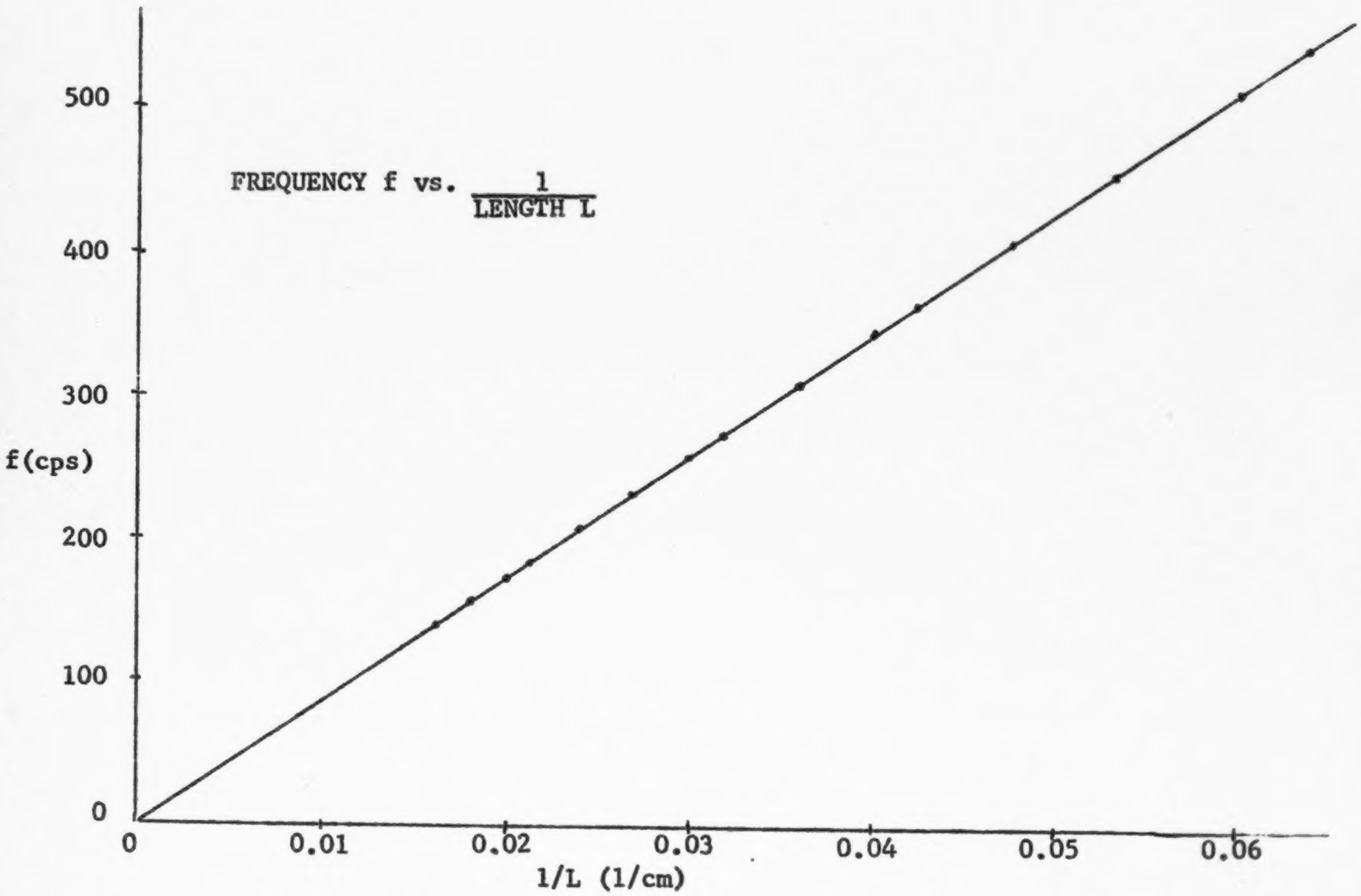
h. Multiplying both sides of the equation by $(1/x)$, this relationship between the variables can be expressed

$$y = k\ (1/x)$$

i. The expression $y = k\ (1/x)$ or its equivalent $xy = k$, is the equation of the curve, the equation of a hyperbola.

j. $y = k\ (1/x)$ is the equation describing INVERSE VARIATION or INVERSE PROPORTION.


ORGAN PIPE EXPERIMENT
FREQUENCY f vs. LENGTH L
f(cps)
500
400
300
200
100
0
0
10
20
30
40
50
60
L (cm)


FREQUENCY f vs. $\frac{1}{\text{LENGTH L}}$
f(cps)
500
400
300
200
100
0
0
0.01
0.02
0.03
0.04
0.05
0.06
1/L (1/cm)

UNIT III. FUNCTIONS, RELATIONS, VARIATION, AND GRAPHS

D. Graphs of data

5. First special case of a CURVE: the HYPERBOLA (continued)

   k. $y = k\ (1/x)$ means that as x increases, y proportionately decreases, e.g., if x is doubled, the value of y is halved; if x increases by a factor of 5467, then y decreases by a factor of 5467.

   l. Contrast INVERSE variation with DIRECT variation.

	DIRECT	INVERSE
(1)	ratio of variables is constant: $y/x = k$.	product of variables is a constant: $xy = k$.
(2)	graph is straight line through origin.	graph is a hyperbola, curve not through origin.
(3)	increasing x results in proportionate increase in y.	increasing x results in proportionate decrease in y.
(4)	described by the equation: $y = kx$.	described by the equation: $y = k\ (1/x)$.

   m. Since many curves have a similar appearance, it is important that we investigate whether or not a given graph of data having the same shape is really a hyperbola of the form

$$y = k\ (1/x).$$

   n. A useful and meaningful way to verify our reasoned guess is to introduce a new variable, z. Let $z = (1/x)$ in a case like this; define z as the RECIPROCAL of x. (In the example, $z = 1/L$)

   o. Compute the values of z from the data (in our example, find the reciprocal of each L value); USE THE RECIPROCAL SCALE ON YOUR SLIDE RULE.

   p. Plot y vs z (in our example, plot f vs $1/L$); examine the y vs z graph.

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1. Plot the data obtained in the class demonstration experiment with the organ pipe. (Frequency as a function of length)
   a. Form the product of the variables at ten different points along the curve.
   b. What can be concluded about the products? What are the units of the products?
   c. Plot a graph of the frequency vs the reciprocal of the length.
   d. From this graph write the relationship between the frequency and the length.

2. Graph the function y - 16/x for integral values of x from -16 to +16.
   a. Tabulate the reciprocal of each x value.
   b. Plot y vs 1/x.
   c. Find the slope of this graph.

3. The functional relation between y and x is given by the following ordered number pairs:

x	x
-7.5	0.98
-6.0	1.22
-4.5	1.63
-3.0	2.34
-1.5	4.88
2.0	-3.67
4.0	-1.83
5.0	-1.47
7.0	-1.05

   a. Plot y vs x.
   b. What relation between y and x does the shape of the graph suggest?
   c. Check your guess mathematically. Be sure to show all your work clearly, including graphs, computed slopes, etc.
   d. Write the correct expression relating y and x.

4. Water drops of various sizes were investigated to determine the pressure P due to surface tension as a function of the radius r of the drops. The following data was obtained for drops in air at a temperature of 25°C:

r (millimeters)	P (dynes/cm^2
0.060	240.
0.120	119.6
0.180	79.8
0.240	59.8
0.300	47.9
0.360	39.9
0.420	34.2

   a. Plot and analyze the data.
   b. Does this kind of relation between P and r make any physical sense? Comment.

5. Examine your graph of data obtained in the pinhole camera experiment. IN LAB BOOK.
   a. What relation between the variables does the shape of the graph suggest?
   b. Form the product of the variables ($h_i d_o$) for each data point. Find the largest percent difference.
   c. Let $z = 1/d_o$ and plot a graph of h_i vs z.
   d. Compute the slope of the resulting line; from this graph, write the expression relating h_i and d_o.

6. The functional relation between y and x is given by the following ordered number pairs:

x	y
0.0	undefined
1.0	15.0
1.5	6.67
2.0	3.75
2.5	2.40
3.0	1.67
3.5	1.23

   a. Plot y vs x.
   b. What relation between y and x does the shape of the graph suggest?
   c. Check your guess mathematically. What can be concluded?

D. Graphs of data

5. First special case of a CURVE: the HYPERBOLA (continued)

   q. If the graph of y vs z is a STRAIGHT LINE THROUGH THE ORIGIN,

      (1) the equation of the line can be written $y = mz$, where m is the slope of the line.

      (2) compute the slope of the line: $m = \Delta y / \Delta z$; insert the value of m thus computed into the expression $y = mz$.

      (3) since $z = 1/x$, substitute into the equation of the y vs z line and write $y = m\ (1/x)$.

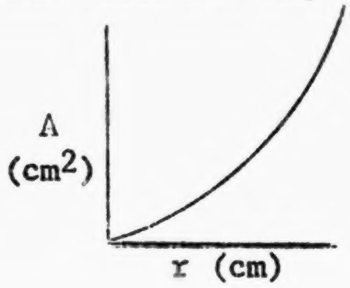
      (4) since m is a constant, $y = m\ (1/x)$ is certainly of the form describing a hyperbola, $y = k\ (1/x)$.

      (5) our reasoned guess is correct and $k = m$ (in our example, the slope of the y vs z graph gives us the same constant found before, $k = 8.24 \times 10^3$ cm/sec).

   r. If the graph of y vs z is not a straight line through the origin (both conditions must be satisfied), the curve cannot be described by the equation $y = k\ (1/x)$; it is not this special case of a hyperbola --- PUNT! (for the time being...)

6. Second special case of a CURVE

   a. Data from LITTLE CIRCLE Experiment: determination of the area of a circle as a function of the radius: A vs r. EXPERIMENT *
      (Alternatively, the mass vs radius data for cylinders of the same material and height but different diameters may be substituted) EXPERIMENT *



   b. The resulting graph is nonlinear and passes through the origin.

   c. This curve is an example of a PARABOLA (one branch obtained here)

Experiment: BIG CIRCLES AND LITTLE CIRCLES (Part 2)

This exercise with small circles (10-cm radius and less) is designed primarily to generate a parabola to be used as an example of the $y = k x^2$ relationship. The methods employed, however, have merit of their own and add to the value of the experiment.

The problem posed in discussion is how to measure accurately (and not compute by formula) the area of circles having different radii. Counting squares on graph paper is easily dismissed as not only approximate but cumbersome as well --- in fact, the less approximate the result, the more cumbersome the process (an excellent example of inverse variation!) The same is true of geometrical (limiting polygon) notions. It takes little for the instructor to guide the discussion toward the "pancake procedure", called by that name merely because the mention of "pancake" has almost assuredly evoked some modification of the method suggested and described below:

(1) Each student or group needs two pieces (notebook size) of the same stiff cardboard, the kind from back of pads or shoe boxes, or the like.

(2) On one, draw with a compass concentric circles of radii r (say) 2.0 cm, 3.5 cm, 5.0 cm, 6.5, 9.0 cm, 9.0 cm, 10.0 cm.

(3) Mark the second cardboard into squares with sides of (say) 20.0 cm, 15.0 cm, 10.0 cm, 5.0 cm, 2.0 cm.

(4) Determine the mass of each square on a laboratory balance, starting with the largest (cut the cardboard for successively smaller squares).

(5) Plot the mass (in grams) of each square vs the area (in cm^2).

(6) Measure the mass m (grams) of each circle on a laboratory balance, starting with the largest (again cut for successively smaller circles).

(7) Since the squares and circles are of the same cardboard, use the graph obtained in (5) to determine the area A (cm^2) of each circle.

(8) Plot the area A (cm^2) of the circles as a function of the radius r (cm). (See class notes and problem sheet for analysis and application)

If time and circumstances permit, an additional graph, the circumference C vs the radius r, may also be obtained (as in the BIG CIRCLE part); C is measured here by merely rolling the circles along a meter stick.

D. Graphs of data

6. Second special case of a CURVE (continued)

d. Note from the data and the graph, for equal changes in r, A increases sharply for larger values of r:

(1) as r changes from 1.0 cm to 2.0 cm, A increases from 3.14 cm^2 to 12.6 cm^2; for Δr = 1.0 cm, ΔA = 9.40 cm^2.

(2) as r changes from 5.0 cm to 6.0 cm, A increases from 78.5 cm^2 to 113. cm^2; for Δr = 1.0 cm, ΔA = 34.5 cm^2.

(3) as r changes from 7.0 cm to 8.0 cm, A increases from 154 cm^2 to 200 cm^2; for Δr = 1.0 cm, ΔA = 46.0 cm^2.

e. The relationship between the quantities here is an example of DIRECT VARIATION WITH THE SQUARE; in the example, A varies directly with the SQUARE of r; this is generally stated: y varies directly with the square of x.

f. Note that the ratio of the area to the square of the radius is the same for every point along the curve:

$$A_1/r_1^2 = A_2/r_2^2 = A_3/r_3^2 = \ldots = \text{constant}$$

g. In general, for this kind of curve, $y/x^2 = k$, where k is a constant.

h. Multiplying through by x^2, this can also be expressed $y = k\,x^2$; $y = k\,x^2$ is the equation of the curve, $y = k\,x^2$ describes this kind of parabola.

i. $y = k\,x^2$ is also the equation describing DIRECT VARIATION WITH THE SQUARE.

j. $y = k\,x^2$ means that as x increases, y increases by the square of x, e.g., if x increases by a factor of 3, y increases by a factor of 9; if x increases by a factor of 39, y increases by a factor of 39^2 or 1521.

k. To verify whether or not a given graph of data is a parabola of this mathematical description, $y = k\,x^2$, introduce a new variable z; in cases like this, $z = x^2$.

l. In the example, $z = r^2$; USE THE A (or B) SCALE OF YOUR SLIDE RULE FOR COMPUTING SQUARES DIRECTLY.

m. Plot y vs z; in the example, plot A vs r^2.

n. Examine the graph of y vs z.

o. If y vs z gives a straight line through the origin,

(1) the equation of the line can be written $y = mz$, where m is $\Delta y/\Delta z$, the slope of the y vs z graph; in the example, the slope of the line is $\Delta A/\Delta r^2$.

(2) since $z = x^2$, write $y = m\,x^2$.

(3) m is the slope of the line and hence is constant; therefore, $y = m\,x^2$ is certainly of the form $y = k\,x^2$, the equation of the parabola.

(4) our reasoned guess is correct and $k = m$.

p. If y vs z is not a straight line through the origin, the data is not of the form $y = k\,x^2$.

QUANTITATIVE SCIENCE DATE________________ NAME________________________

1. Examine your graph of data obtained in the experiment in which the Area A of a circle was measured as a function of the radius r.

    a. What relation between the variables does the shape of the curve suggest?

    b. Form the ratio A/r^2 for each data point. What can be concluded about these products? What are the units of these products?

    c. Let $z = r^2$ and plot a graph of A vs z.

    d. Compute the slope of the resulting line; from this graph, write the expression relating A and r.

2. Graph the expression $y = 0.25\ x^2$ for integral values of x from -5 to +5.

    a. Tabulate the square of each x value.

    b. Plot y vs x^2.

    c. Find the slope of this graph. Conclude.

3. The relation between y and x is given by the following ordered number pairs:

x	y
0	0
2	444
4	1778
6	3998
8	7100

    a. Plot y vs x.

    b. What relation between y and x does the shape of the curve suggest?

    c. Check your guess mathematically.

    d. Write the correct expression relating y and x after analyzing the data.

4. On one graph, plot the three relations: $y = 2\ x^2$, $y = \frac{1}{2}\ x^2$, and $y = -\ \frac{1}{2}\ x^2$ for integral values of x from -5 to +5. Discuss the effect of the sign and the size of the constant multiplying x^2 in each case, i.e., the COEFFICIENT of x^2.

QUANTITATIVE SCIENCE DATE______________ NAME________________________

The mass of a square of cardboard 8.0 cm on a side was measured and found to be 22.0 grams. Five regular geometrical figures of varying size were cut from identical cardboard. The BASE b and MASS m of each figure was measured and gave the following results:

b (cm)	m (grams)
5.0	4.30
7.5	21.8
10.0	68.8
12.5	168.
15.0	355.

Use this data to construct a graph of the AREA A as a function of the BASE b of these figures.

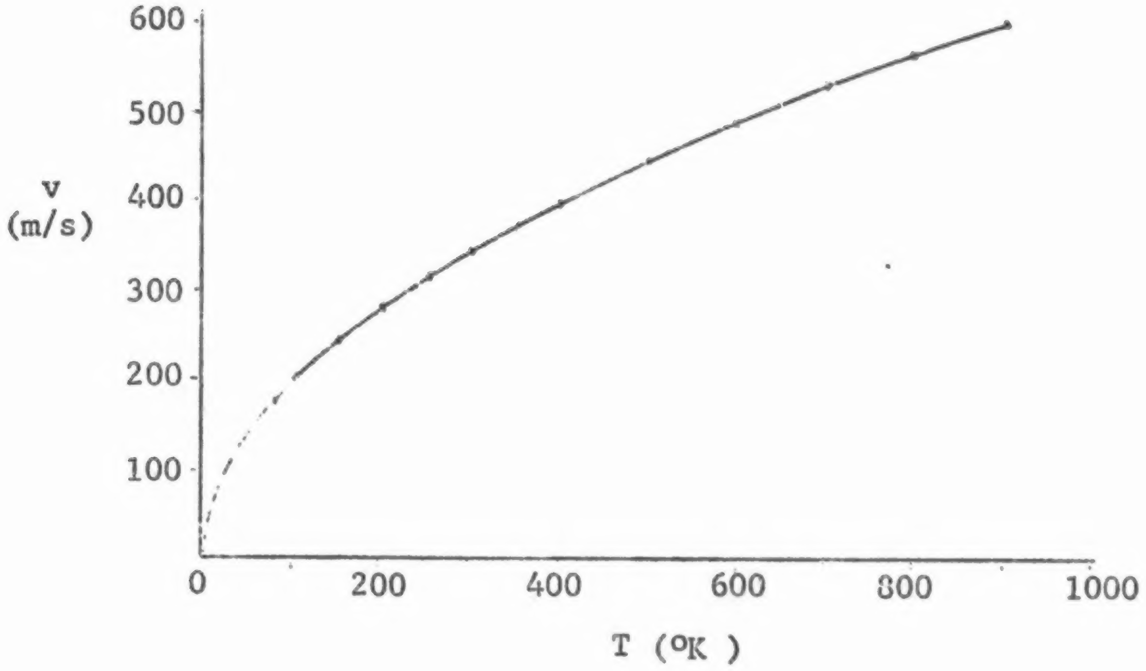
UNIT III. FUNCTIONS, RELATIONS, VARIATION, AND GRAPHS

D. Graphs of data (continued)

7. Third special case of a CURVE

a. EXAMPLE: Speed of sound in air as a function of the absolute temperature; v vs T

Temperature T (°K)	Speed v (meters/second)
100	201
150	245
200	283
250	316
300	347
350	374
400	401
500	448
600	491
700	531
800	570
900	600



(While the purpose in introducing this data is merely to illustrate a parabola opening on the x-axis, a brief reference to the physical situation is quite in order. First remarks may be directed toward the Kelvin temperature scale ((-273° C = 0 °K; certainly everybody has heard about absolute zero!)). At temperatures much below 100 °K, the physical situation changes considerally ((at

D. Graphs of data

7. Third special case of a CURVE (continued)

atmospheric pressure, oxygen is a liquid below 90 °K; nitrogen is a liquid below 77 °K)) so no data is given for the gas in this temperature range. A dashed line on the drawn graph indicates the extension of the mathematical description beyond the limits of the range of data.)

(The graph of the period as a function of the length for the pendulum may also be used as an example of the type of variation discussed in this section)

EXPERIMENT *

b. The resulting graph is nonlinear and (extended) passes through the origin.

c. This curve is an example of a special parabola, one opening on the x-axis.

d. Note from the data and the graph, for each equal change in the temperature T, the increase in the speed v gets smaller and smaller at higher T values:

> As T changes from 100 °K to 200 °K, v increases from 201 m/s to 283 m/s; for a ΔT = 100 °K, Δv = 82 m/s;
> As T changes from 800 °K to 900 °K, v increases from 570 m/s to 600 m/s; for ΔT = 100 °K, Δv = 30 m/s.

e. The relationship between the quantities here is an example of one quantity varying directly with the square root of another; in the example, v varies directly with the square root of T; this is generally stated, y varies directly with the square root of x, or y is directly proportional to the square root of x.

f. Note that the ratio $v/\sqrt{T}$ is the same for every point along the curve:

$$v_1/\sqrt{T_1} = v_2/\sqrt{T_2} = v_3/\sqrt{T_3} = \ldots = \text{constant}$$

(see the supplementary sheet for instructions on using the A scale of the slide rule for finding square roots)

g. In general, for this kind of curve, $y/\sqrt{x} = k$, where k is a constant.

h. Multiplying both sides by $\sqrt{x}$, the expression describing direct variation with the square root, or the equation of the square root curve is obtained:

$$y = k\sqrt{x}$$

i. $y = k\sqrt{x}$ means that as x increases, y increases by the square root of x: if x increases by a factor of 2, y increases by a factor of $\sqrt{2}$ or 1.41; if x increases by a factor of 75, y increases by a factor of $\sqrt{75}$ or 8.65.

The Slide Rule - Square Roots

To find the square root of a number:

If the number is greater than one and written in the usual decimal form (for example, 3.85, 74), count the number of digits to the left of the decimal point (in the number 3.85, there is one digit to the left of the decimal point; in the number 74.0, there are two digits to the left of the decimal point)

→if the number of digits to the left of the decimal point is ODD (1,3,5,7,...) use the LEFT HALF of the A scale

→if the number of digits to the left of the decimal point is EVEN (2,4,6,8,...) use the RIGHT HALF of the A scale

Set the indicator hairline at the number whose square root is desired on the A scale; read the square root directly below on the D scale. Supply the decimal point by the nearest known square roots.

Example: $\sqrt{3.85}$ Since there is one digit to the left of the decimal point use LEFT half of A scale. Set indicator hairline at 385 on left half of A scale; read 1961 on D scale below. Since $1^2 = 1$ and $2^2 = 4$, the square root of 3.85 lies between 1 and 2. Proper decimal placement gives 1.961.

Example: $\sqrt{74.0}$ Since there are two digits to the left of the decimal point use RIGHT half of A scale. Set indicator hairline at 740 on right half of A scale; read 860 on D scale below. Since 8^2 is 64 and 9^2 is 81, the square root of 74.0 lies between 8 and 9. Proper decimal point placement gives 8.60.

Using powers-of-ten notation can greatly simplify the process of extracting square roots of numbers on the slide rule because it applies to all numbers, fractional and whole, large or small.

Write the number whose square root is desired with either one or two digits to the left of the decimal point depending on which choice will give an EVEN exponent in the power of ten ("Modified" Scientific Notation).

Examples: $53{,}000 = 5.30 \times 10^4$ $\quad 0.176 = 17.6 \times 10^{-2}$

$2950 = 29.50 \times 10^2$ $\quad 0.000755 = 7.55 \times 10^{-4}$

In this process, the square root of the numerical part and the square root of the power of ten part are found separately.

→the square root of the numerical part is obtained by setting the indicator hairline at the number on the A scale and reading the root below the hairline on the D scale. If there is one digit in the numerical part to the left of the decimal point, use the LEFT half of the A scale; if there are two digits to the left of the decimal point in the numerical part, use the RIGHT half of the A scale.

→the square root of the power of ten is obtained by dividing the exponent by two

Example: $\sqrt{5.30 \times 10^4}$ Use left half of A scale since there is one digit to left of the decimal point. Set the indicator hairline at 5.30 on the left half of the A scale; read 2.30 on the D scale. The square root of 10^4 is 10^2 (obtained by dividing the exponent by two). So the square root of 5.30×10^4 (or 53,000) is 2.30×10^2 (or 230).

Example: $\sqrt{0.176} = \sqrt{17.6 \times 10^{-2}}$ Set the indicator hairline at 17.60 on the Right half of the A scale; read 4.195 on the D scale. The square root of 10^{-2} is 10^{-1}. So desired square root is 4.195×10^{-1} (or 0.4195).

QUANTITATIVE SCIENCE DATE________________ NAME________________________

FIND THE SQUARE ROOTS OF THE FOLLOWING NUMBERS ON YOUR SLIDE RULE:

1. 1.44 1________________

2. 4.41 2________________

3. 6.75 3________________

4. 9.50 4________________

5. 14.4 5________________

6. 27.0 6________________

7. 35.5 7________________

8. 56.4 8________________

9. 70.0 9________________

10. 95.0 10________________

USE MODIFIED SCIENTIFIC NOTATION IN FINDING THE SQUARE ROOTS OF THE FOLLOWING NUMBERS ON YOUR SLIDE RULE:

11. 144 11________________

12. 635,000 12________________

13. 0.332 13________________

14. 14,400 14________________

15. 2,220,000 15________________

16. 0.0875 16________________

17. 144,000 17________________

18. 0.000720 18________________

19. 1440 19________________

20. 0.0000134 20________________

QUANTITATIVE SCIENCE DATE________________ NAME____________________________

USE MODIFIED SCIENTIFIC NOTATION AND YOUR SLIDE RULE TO DETERMINE THE CUBE ROOTS OF THE FOLLOWING NUMBERS:

1. 8.11 1.____________________

2. 3.72 2.____________________

3. 29.4 3.____________________

4. 128 4.____________________

5. 582 5.____________________

6. 76,500 6.____________________

7. 4460 7.____________________

8. 151,000 8.____________________

9. 9,150,000 9.____________________

10. 0.222 10.____________________

11. 0.000965 11.____________________

12. 0.00270 12.____________________

13. 0.0167 13.____________________

14. 0.147 14.____________________

15. What is the radius of a solid aluminum sphere whose mass is 11.31 grams?

15.____________________

QUANTITATIVE SCIENCE DATE____________________ NAME____________________

Use your slide rule in performing the required operations. In the space provided SHOW CLEARLY how the decimal point (or power of ten) is determined.

FIND THE RECIPROCALS OF THE FOLLOWING NUMBERS:

1. 96.5 1____________________

2. 0.311 2____________________

3. 2380 3____________________

4. 0.00560 4____________________

FIND THE SQUARE ROOTS OF THE FOLLOWING NUMBERS:

5. 810 5____________________

6. 2560 6____________________

7. 0.196 7____________________

8. 0.001681 8____________________

FIND THE CUBE ROOTS OF THE FOLLOWING NUMBERS:

9. 1,250,000 9____________________

10. 4.32 10____________________

11. 0.27 11____________________

12. 0.0685 12____________________

PERFORM THE INDICATED OPERATIONS:

13. $(472)^3$ 13____________________

14. 1/247 14____________________

15. $(0.274)^2$ 15____________________

QUANTITATIVE SCIENCE DATE______________ NAME________________________

1. Plot the data obtained in the class example on the speed v (meters/sec) of sound in air as a function of the absolute temperature T (^oK).
   a. Use the slide rule to find the square root of each temperature value.
   b. Form the ratio $v/\sqrt{T}$ for each data point.
   c. What can be concluded about the computed ratios? What are the units of the ratios?
   d. Plot a graph of speed vs the square root of the temperature.
   e. From this graph, write the relationship between v and T, complete with units.

2. Examine your graph of the period T(sec) vs the length L(cm) obtained in the pendulum experiment. IN LAB BOOK.
   a. What relation between the variables does the shape of the graph suggest?
   b. Use your slide rule to find the square root of ten distributed length values.
   c. Form the ratio of $T/\sqrt{L}$ for the ten data points. What are the units of these ratios?
   d. What can be concluded about these ratios? Express the error quantitatively*
   e. Plot a graph of the period vs the square root of the length.
   f. From this graph, write the relationship between T and L, complete with units.

3. An experiment was designed to measure the speed v (ft/sec) as a function of the distance S (ft) an automobile travels while accelerating from rest.

S (feet)	v (feet/second)
0	0
30	15.9
60	22.5
90	27.6
120	31.8
150	35.7

Plot and analyze the data.

4. Plot the function $y = 6.35\sqrt{x}$, for half-integral values of x from 0.5 to +5.0.
   a. From the graph: as x changes from 0.75 to 1.75, what is the corresponding change in y?
   b. From the graph: as x changes from 3.75 to 4.75, what is the corresponding change in y?
   c. Compare the Δx's; compare the Δy's.
   d. Comment on NEGATIVE values of x.

QUANTITATIVE SCIENCE DATE______________ NAME________________________

Recall the "MASSER" used earlier in the discussion of inertia. Using a similar device, the following data for the period T as a function of the mass m was obtained:

T(seconds)	m(patties)
0.315	0.25
0.445	0.50
0.630	1.0
0.880	2.0
1.09	3.0
1.26	4.0
1.41	5.0
1.54	6.0

Plot and analyze the data. WRITE THE FINAL EXPRESSION (COMPLETE WITH UNITS) ON THE LINE BELOW.

____

QUANTITATIVE SCIENCE DATE______________ NAME______________________

An agriculture expert conducted an experiment to measure the time t (hours) required to plant a certain number of acres of corn as a function of the number n (men) workers. Plot and analyze the data. WRITE THE FINAL EXPRESSION (COMPLETE WITH UNITS) ON THE LINE BELOW.

n(men)	t(hours)
3	5.90
5	3.54
7	2.53
9	1.97
11	1.61

____

An experiment was designed to measure the speed v (feet/second) as a function of the accelerating distance S (feet) for an automobile starting from rest. Plot and analyze the data. WRITE THE FINAL EXPRESSION (COMPLETE WITH UNITS) ON THE LINE BELOW.

S(feet)	v(feet/second)
0	0
30	15.9
60	22.5
90	27.6
120	31.8
150	35.7

____

3

UNIT III. FUNCTIONS, RELATIONS, VARIATION, AND GRAPHS

D. Graphs of data

7. Third special case of a CURVE (continued)

   j. To verify whether or not a given graph of data is a square root curve, introduce a new variable z; in a case like this, let $z = \sqrt{x}$; in the example, $z = \sqrt{T}$

   k. Compute the values of z from the data (or the graph)

   l. Plot y vs z; examine the resulting graph.

   m. If y vs z gives a straight line through the origin,

      (1) the equation of the line can be written, $y = mz$, where m is the slope of the y vs z line;

      (2) compute the slope of the line from the graph; insert this value for m in $y = mz = m\sqrt{x}$, since $z = \sqrt{x}$.

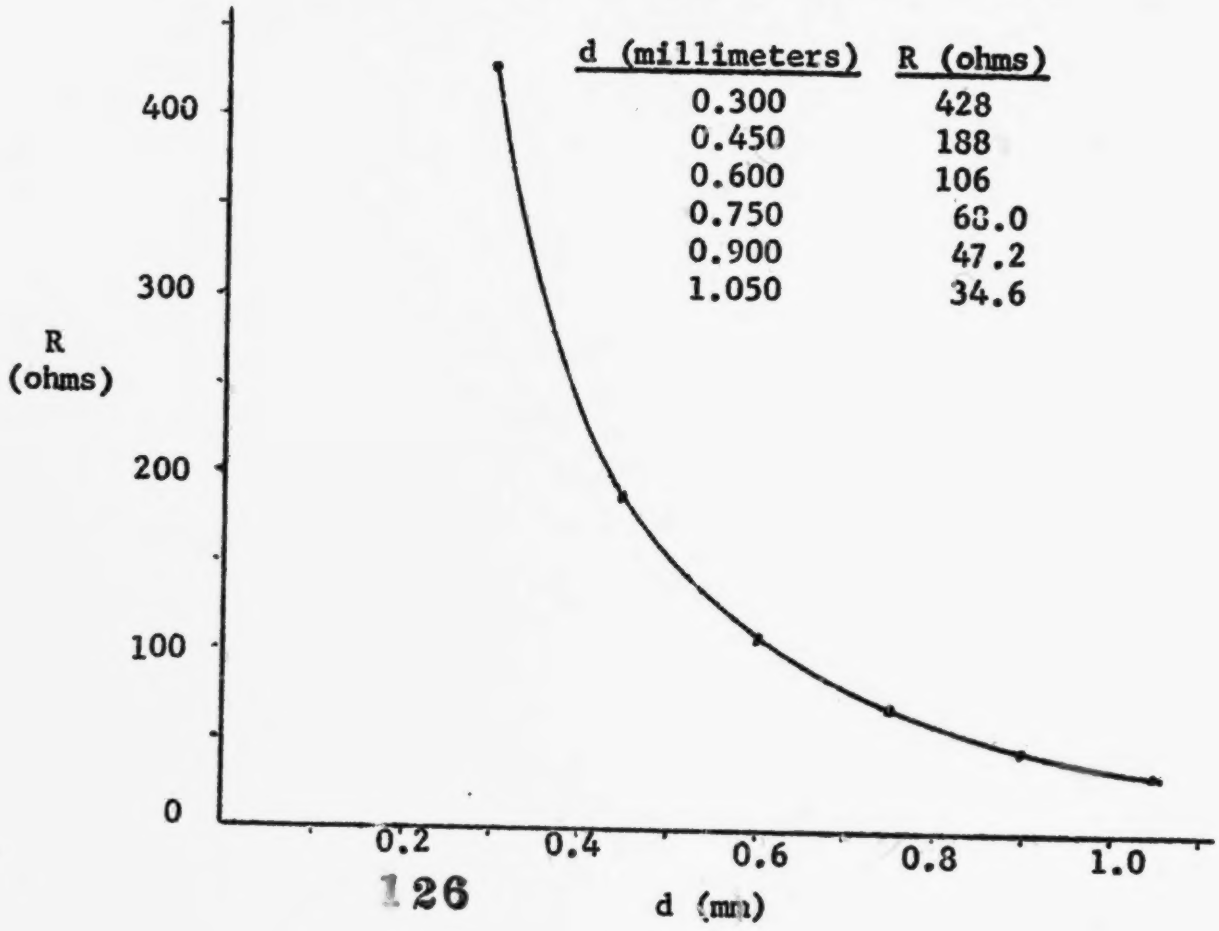
      (3) m is the slope of a line and therefore constant; hence $y = m\sqrt{x}$ is certainly of the form $y = k\sqrt{x}$.

      (4) Our reasoned guess is correct and k is equal to m.

   n. If y vs z is not a straight line through the origin, the curve and data cannot be described by the equation $y = k\sqrt{x}$.

8. Fourth special case of a CURVE

   a. EXAMPLE: Electrical resistance of a given length of nichrome wire as a function of the wire diameter: R vs d



d (millimeters)	R (ohms)
0.300	428
0.450	188
0.600	106
0.750	68.0
0.900	47.2
1.050	34.6

QUANTITATIVE SCIENCE DATE________________ NAME______________________

1. Plot the data given in class for the electrical resistance R (ohms) of a Nichrome wire as a function of the wire diameter d (millimeters).

   a. Form the product Rd^2 for each of the data points.

   b. What can be concluded about the products? What are the units of the products?

   c. Let $z = 1/d^2$ and plot R vs z.

   d. From the graph, write the expression relating R and d.

2. Examine your graph of data obtained in the experiment on the intensity of illumination.

   a. What relation between the variables does the shape of the graph suggest?

   b. Choose the z that corresponds to your reasoned guess of part (a) above and plot I vs z.

   c. Is drawing a straight line through the origin justified? What percent error would this indicate?

   d. Assume the validity of representing the plotted points by a straight line through the origin; from the graph, write the complete expression relating I and r.

3. Refer to number 6 on the problem sheet dealing with inverse variation, $y = k(1/x)$. Test the points for inverse square variation.

D. Graphs of Data

8. Fourth special case of a CURVE: INVERSE SQUARE (continued)

b. The resulting graph is nonlinear and does not pass through the origin.

c. Although the curve resembles a hyperbola in general appearance, it has a different mathematical description (the product of the variables is not a constant).

d. Note from the data and from the graph, for each equal increase in d, R decreases less and less at greater d values; as d is increased from 0.30 mm to 0.45 mm, R decreases from 428 ohms to 188 ohms, or for a $\Delta d = 0.15$ mm, the corresponding $\Delta R = 240$ ohms; as d is increased from 0.90 mm to 1.05 mm, R decreases from 47.2 to 34.6 ohms, or a for a $\Delta d = 0.15$ mm, the corresponding $\Delta R = 12.6$ ohms.

e. The relationship between the quantities here is an example of one quantity varying inversely with the square of another; in the example, R varies inversely with the square of d; this is generally stated, y varies inversely with the square of x, or y is inversely proportional to the square of x.

f. Note that the product Rd^2 is the same for every point along the curve:

$$R_1d_1^2 = R_2d_2^2 = R_3d_3^2 = \dots = \text{constant}$$

g. In general, for this kind of curve, $yx^2 = k$, where k is a constant.

h. Multiplying both sides by $(1/x^2)$, $y = k\ (1/x^2)$; this expression, or its equivalent $yx^2 = k$, is the equation of the INVERSE SQUARE CURVE; $y = kx^2$ is also the equation describing inverse variation with the square.

i. $y = k\ (1/x^2)$ means that as x increases, y DECREASES by the SQUARE of x:

if x increases by a factor of 2, y decreases by a factor of 4;
if x increases by a factor of 13, y decreases by a factor of 169.

j. To verify whether or not a given graph of data gives an inverse square relation, introduce a new variable z; in a case like this, let $z = (1/x^2)$; in the example, $z = (1/d^2)$.

k. Compute the values of z from the data (or graph).

l. Plot y vs z: examine the resulting graph.

m. If y vs z gives a straight line through the origin,
  (1) the equation of the line can be written $y = mz$, where m is the slope of the y vs z line;
  (2) compute the slope of the line; insert this value for m in $y = mz = m\ (1/x^2)$, since $z = (1/x^2)$;
  (3) m is the slope of the line and therefore constant; hence, $y = m\ (1/x^2)$ is certainly of the form $y = k\ (1/x^2)$;
  (4) our reasoned guess is correct and $k = m$.

n. If y vs z is not a straight line through the origin, the curve and data cannot be described by the equation $y = k\ (1/x^2)$.

QUANTITATIVE SCIENCE DATE__________ NAME____________________

TWO SMALL METAL SPHERES WHEN CHARGED ELECTRICALLY ARE FOUND TO REPEL EACH OTHER WITH A FORCE F, WHICH DEPENDS ON THE DISTANCE r BETWEEN THE CENTERS OF THE SPHERES. PLOT AND ANALYZE THE FOLLOWING DATA. WRITE THE FINAL EXPRESSION, COMPLETE WITH UNITS, ON THE LINE BELOW.

F (dynes)	r (centimeters)
30.8	9.0
17.4	12.0
7.73	18.0
4.34	24.0
2.78	30.0
1.93	36.0

UNIT III. FUNCTIONS, RELATIONS, VARIATIONS, AND GRAPHS

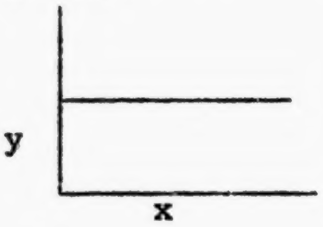
D. Graphs of data (continued)

9. Summary of "Recognition Method" of analyzing data

   a. Start with data: experimental measure of two related quantities, the independent variable x and the dependent variable y.

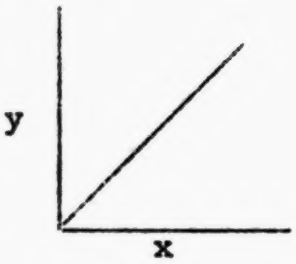
   b. Plot y vs x; completely identify axes (numerical values and units).

   c. If the graph of y vs x is a straight line parallel to the x axis:



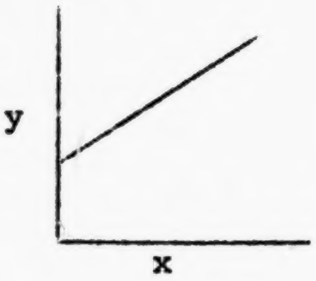
y does not depend on x: y is a constant function; equation of line, $y = k$, where k is a constant; slope of this line is zero.

   d. If the graph of y vs x is a straight line through the origin:



y varies directly with x;
ratio y/x is a constant;
expression of line, $y = kx$, where
k is a constant.
slope of the line, $\Delta y/\Delta x$, gives the value of k, so $y = mx$.

   e. If the graph of y vs x is a straight line not through the origin:



equation of the line is $y = mx + b$, where m is the slope and b is the y-intercept; compute m by $\Delta y/\Delta x$; find b by inspecting graph.

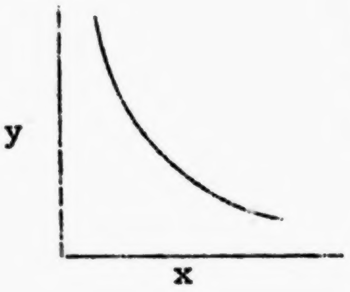
UNIT III. FUNCTIONS, RELATIONS, VARIATION, AND GRAPHS

D. Graphs of data

9. Summary of "Recognition Method" of analyzing data (continued)

f. If the graph of y vs x is a curve, FROM THE SHAPE

(1)



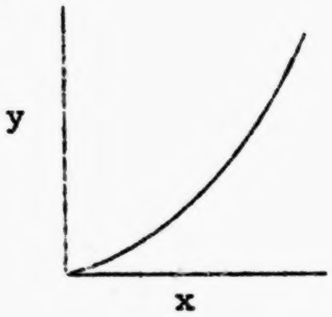
GUESS: $y = k\ (1/x)$

VERIFY: let $z = (1/x)$

PLOT: y vs z; if straight line through origin, slope gives k.

EQUATION: $y = k\ (1/x)$, INVERSE VARIATION

(2)



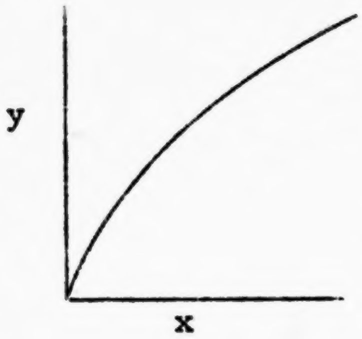
GUESS: $y = k\ x^2$

VERIFY: let $z = x^2$

PLOT: y vs z; if straight line through origin, slopegives k.

EQUATION: $y = k\ x^2$, DIRECT VARIATION WITH THE SQUARE

(3)



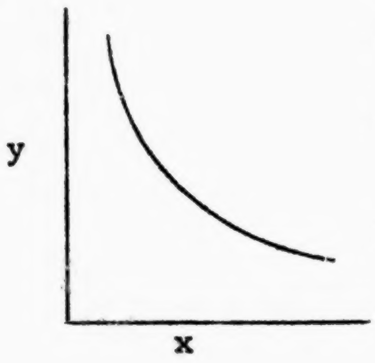
GUESS: $y = k\sqrt{x}$

VERIFY: let $z = \sqrt{x}$

PLOT: y vs z; if straight line through origin, slope gives k.

EQUATION: $y = k\sqrt{x}$, DIRECT VARIATION WITH THE SQUARE ROOT.

(4)



GUESS: $y = k\ (1/x^2)$

VERIFY: let $z = (1/x^2)$

PLOT: y vs z; if straight line through origin, slope gives k.

EQUATION: $y = k\ (1/x^2)$, INVERSE SQUARE VARIATION

QUANTITATIVE SCIENCE DATE________________ NAME______________________

Assume the sets of ordered pairs in each case represent data (measured values) obtained in an experiment. ANALYZE each set of data. The first column in each case represents the INDEPENDENT VARIABLE.

1.

x(clydes)	y(finks)
0.008	0.55
0.020	0.87
0.040	1.23
0.080	1.74
0.120	2.13
0.160	2.46
0.200	2.75
0.240	3.01

1____

2.

u(oomphs)	v(toots/bazook)
1.00	5.24
3.00	6.90
5.00	8.57
7.00	10.25
9.00	11.91
11.0	13.58

2____

3.

m(beatles/aardvark)	n(chimp grombies)
150	988
300	247
450	110
600	62
750	38
900	26

3____

4.

p(pankies)	q(hankies)
0	0
1.0	3.48 X 10^{-3}
2.5	4.73 X 10^{-3}
5	5.95 X 10^{-3}
10	7.50 X 10^{-3}
15	8.60 X 10^{-3}
20	9.45 X 10^{-3}

4____

5.

x(storgs)	y(taters)
0.10	9.5 X 10^{-5}
0.20	4.75 X 10^{-5}
0.30	3.17 X 10^{-5}
0.40	2.38 X 10^{-5}
0.50	1.90 X 10^{-5}
0.60	1.58 X 10^{-5}

5____

QUANTITATIVE SCIENCE DATE________________ NAME________________________

Assume the sets of ordered pairs in each case represent data (measured values) obtained in an experiment. ANALYZE each set of data. The first column in each case represents the INDEPENDENT variable.

6.

r(khleides)	s(phynques)
3	10
6	20
9	30
12	40
15	50
18	60

6____

7.

u (millibusters)	v (centigrunts)
0	0
0.5	0.970
1.0	7.75
1.5	26.2
2.0	62.0
2.5	121.
3.0	209.

7.____

8.

x (wallies/kuk)	y (megagobs)
-5	335
-3	201
0	0
+2	-134
+4	-268

8____

* 9.

x (kaniffs)	y(decatugs)
0	0
0.5	4.38
1.0	10.50
1.5	18.38
2.0	28.0
2.5	39.4
3.0	52.5

9____

QUANTITATIVE SCIENCE DATE__________ NAME______________________________

EXPRESS EACH OF THE FOLLOWING BY APPROPRIATE EQUATIONS. NOTE: when the value of a constant is not known or cannot be determined, use the symbol k, e.g., in (2), (5), (6), etc.

1. A straight line through the origin and passing through the point (4,8) 1______________________

2. When x increases 9 times, y increases by a factor of 3 2______________________

3. A straight line crossing the y-axis at +3 and passing through the point (5,7) 3______________________

4. The area A of a circle as a function of the radius r 4______________________

5. The height h_i of a silhouette image as a function of the image distance d_i 5______________________

6. The period T of a pendulum as a function of the bob mass m 6______________________

7. A straight line through the origin and passing through the point (-2,6) 7______________________

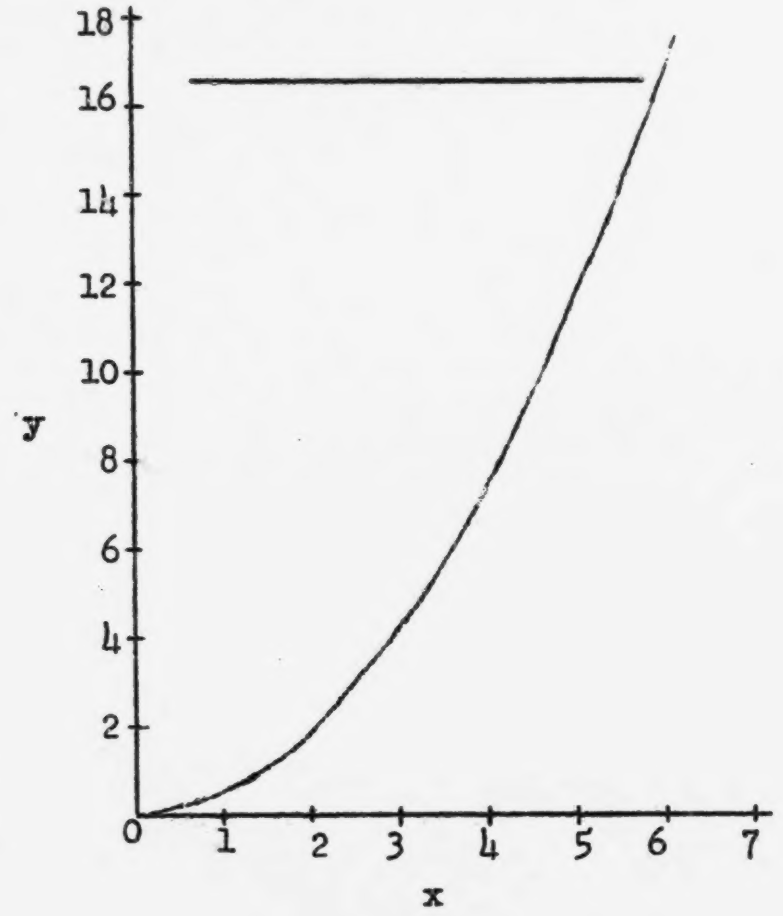
8. The height h_i of a pinhole image as a function of the object distance d_o 8______________________

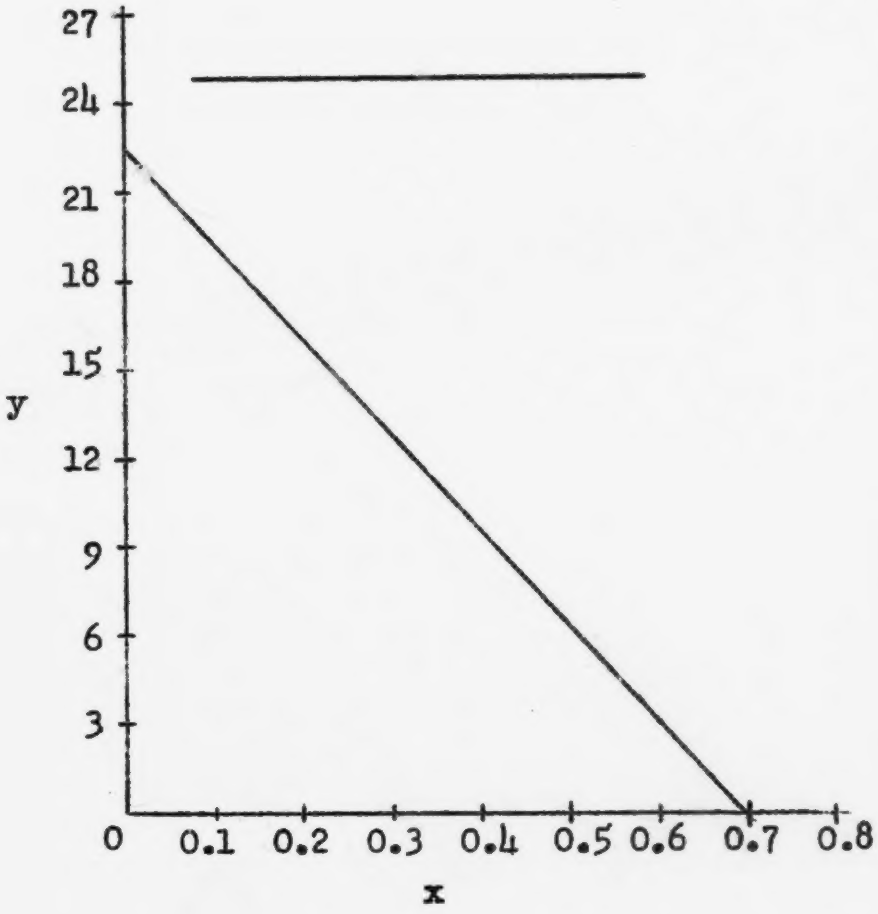
9. A straight line crossing the y-axis at -2 and having a slope of ½ 9______________________

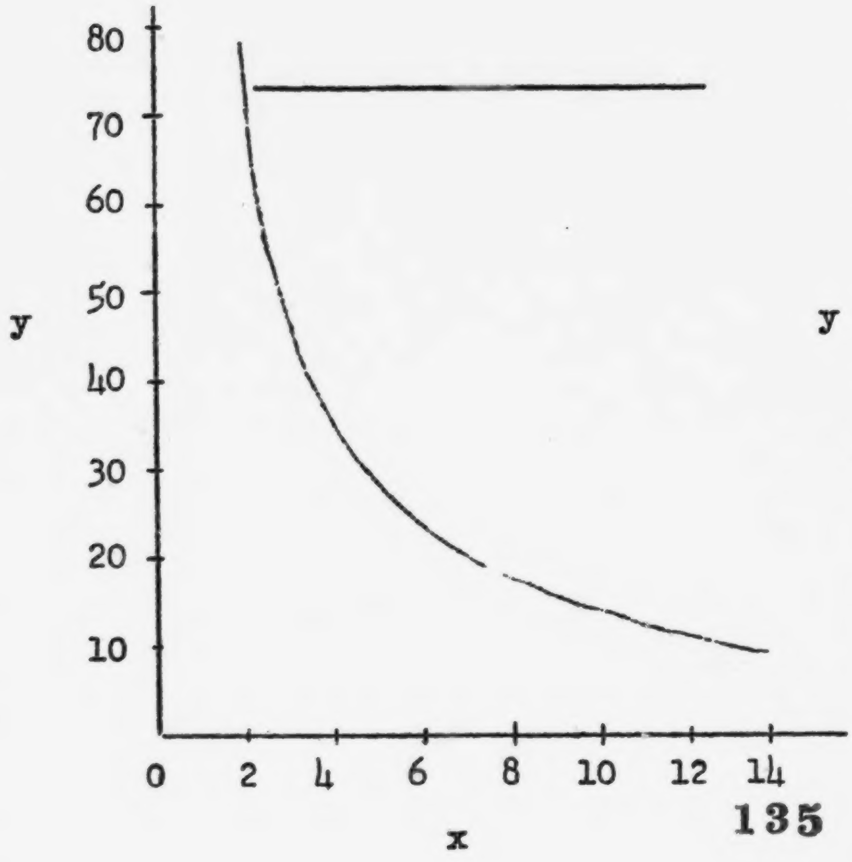
10. A parabola opening on the positive y axis, passing through the origin, and passing through the points (1,3) and (2,12) 10______________________

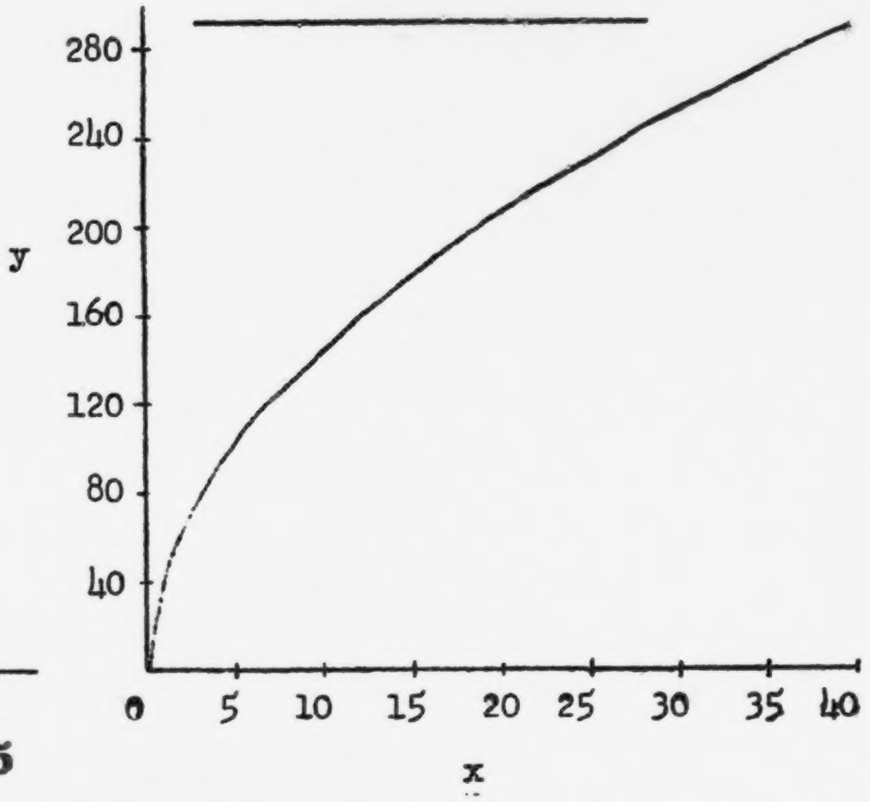
QUANTITATIVE SCIENCE DATE____________ NAME________________________

EXAMINE EACH GRAPH BELOW TO DETERMINE THE RELATION BETWEEN THE VARIABLES (THE GRAPHS ARE DRAWN ON A CENTIMETER SCALE SO THE USE OF A RULER MAY HELP YOU TO IDENTIFY GRAPHICAL VALUES). A COMPLETE ANALYSIS OR VERIFICATION OF YOUR "REASONED GUESS" IS NOT EXPECTED; A CHECK IS HELPFUL AND IN SOME CASES NECESSARY IN THE EVALUATION OF CONSTANTS. WRITE THE FINAL EXPRESSION ON THE BLANK PROVIDED BY THE GRAPH.







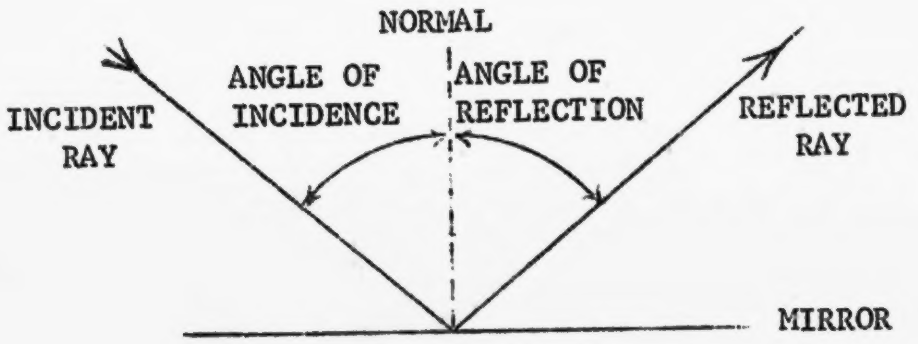


UNIT IV. REFLECTION OF LIGHT

A. Laws of Reflection

1. THE INCIDENT RAY, THE NORMAL, AND THE REFLECTED RAY ALL LIE IN THE SAME PLANE — DEMONSTRATION
    - a. NORMAL: imagined or constructed perpendicular to the surface at the point of incidence (also the point of origin of the reflected ray)
    - b. While experimentally we actually use a pencil of light, it is convenient and consistent to state the laws of reflection using the term "rays" of light.
    - c. It is worthwhile to note that the second law has little meaning without the first.

2. THE ANGLE OF REFLECTION IS EQUAL TO THE ANGLE OF INCIDENCE — DEMONSTRATION



   - a. Note that the angles are measured from the normal.
   - b. It is important to realize that the above laws apply not only to plane surfaces, but to curved surfaces as well.
   - (c. While this equality of angles can easily be shown using only a plane mirror and a collimated flashlight beam, an even more satisfying quantitative result is obtained by using either an Optical Disc or Klinger Blackboard Optics Kit).

B. Types of reflection

1. DIFFUSE (collimated flashlight beam incident on a white cloth or sheet of paper as example): the well-defined incident pencil of light is "scattered", a general glow or halo results. — DEMONSTRATION
2. SPECULAR (light incident on a polished metal surface as example): the reflected pencil of light is just as sharp as the incident pencil. — DEMONSTRATION
    - a. Define MIRROR: any surface from which the reflection is specular.
    - b. Laws established above usually referred to as the Laws of Specular Reflection.

B. Types of Reflection (continued)

3. Examine the surface structure of a diffuse reflector under a microscope. **EXPERIMENT**
   a. Observe the random orientation of the microsurfaces.
   b. Conclude: the diffuse character of the reflected light is due to the angular positions of these microsurfaces, each reflecting specularly.
   c. Hence, despite the different total effect observed, the same laws govern the reflection here.

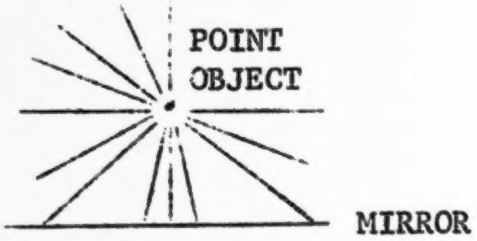
C. Plane mirrors

1. Lab investigation **EXPERIMENT ***
   a. Study qualitative features of the image formed by a plane mirror.
   b. Relate the image distance q and the object distance p (both quantities are measured from the mirror surface).
   c. Relate image size h_i to the object size h_o.
2. Experimental results
   a. The image is apparently behind the mirror, erect, reversed, and cannot be formed on a screen.
   b. The image is located as far behind the mirror as the object is in front of the mirror; since both are measured from the mirror surface, it is convenient to assign one positive values and the other negative, thus,

      (-) image distance = (+) object distance,

      or $q = -p$
   c. The image size is the same as the object size,

      or $h_i = h_o$
3. The laws of specular reflection and image formation by a plane mirror
   a. Light is given off in all directions by a point object; some strike mirror surface.

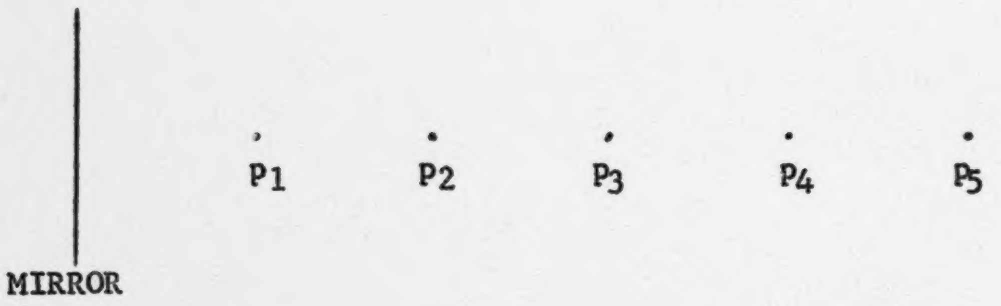


   b. This light is specularly reflected, so the angle of reflection must equal the angle of incidence for each ray striking the mirror.

Experiment: PLANE MIRRORS **INSTRUCTOR INFORMATION SHEET**

The experiment in essence is not unlike the familiar "pins" exercise (see, for example, "Reflection from a Plane Mirror", P.S.S.C. PHYSICS LABORATORY GUIDE), but is designed to obtain a graphical result as well.

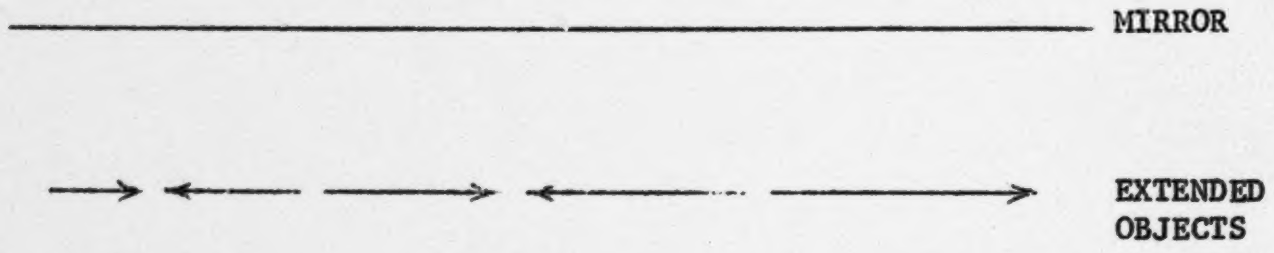
(1) The object pin is placed at 2.0 cm intervals along a line drawn perpendicular to the mirror, giving several different values of the object distance p from the mirror;



(2) The apparent location of the virtual image is found by sighting in the usual way (implicit, of course, is the rectilinear propagation of light); this gives the image distance q from the mirror corresponding to each object distance p;

(3) The result of plotting q vs p is a straight line through the origin, of unit slope.

(4) A series of "extended objects" positioned parallel to the mirror (such as those in the diagram below) can serve to make the image-object size relationship quite apparent:



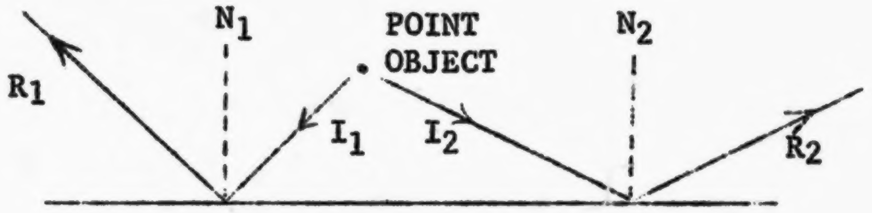
(5) Each extended image is found by sighting on pins placed at the head and tail of each object;

(6) Each image is measured and plotted against the corresponding object size, yielding a straight line through the origin of unit slope, and the conclusion $h_i = h_o$.

C. Plane mirrors

3. The laws of specular reflection and image formation by a plane mirror (continued)

   c. Consider a ray like I_1; draw N_1 and measure the angle of incidence; the reflected ray R_1 leaves the mirror at an angle from the normal equal to the angle of incidence.



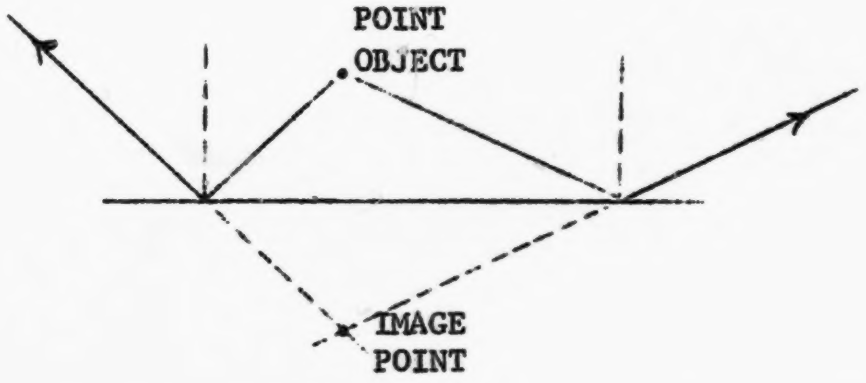
   d. Consider another ray like I_2; draw N_2 and measure the angle of incidence; draw the reflected ray R_2.

   e. The reflected rays, like R_1 and R_2, are the light rays that reach our eyes; we see the light that is reflected from the mirror.

   f. These reflected rays diverge in leaving the mirror; the separation between the rays increases.

   g. This divergence of the light rays reaching our eyes makes the light appear to be coming from a point behind the mirror.

   h. To locate the apparent origin of these diverging rays, extend the reflected rays backward to their point of intersection behind the mirror.



   i. This point of intersection (common to all the reflected rays extended behind the mirror) locates the image point behind the mirror.

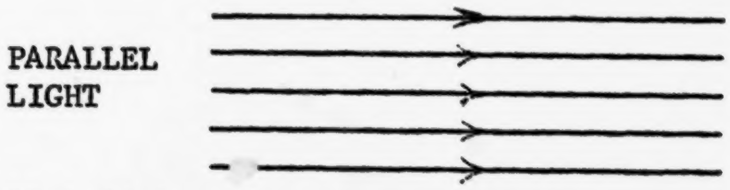
   j. An image thus formed by a plane mirror is called a VIRTUAL IMAGE: no light actually exists at the location of the image; the divergence of the reflected light makes it appear the light is coming from the image position behind the mirror.

   k. Show that the laws of reflection predict the results obtained in the experiment.

UNIT IV. REFLECTION OF LIGHT (continued)

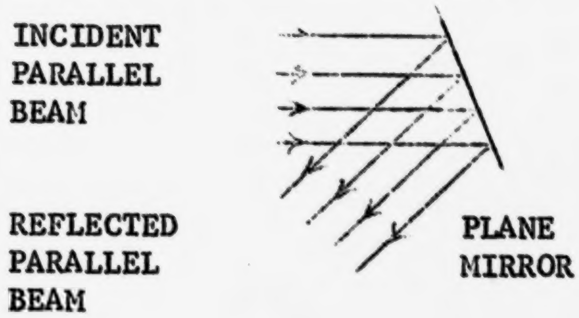
D. Evolution of the converging mirror

1. PROBLEM: Given a beam of parallel light; is it possible to converge all the light to a single point by reflection?



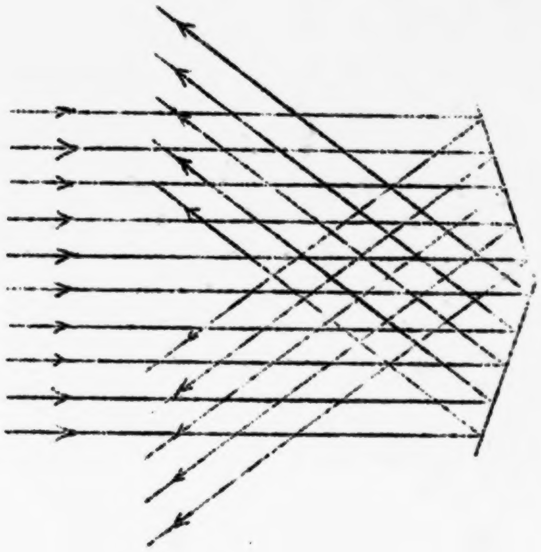
2. Parallel light and the plane mirror

   a. A single plane mirror does not converge parallel light. DEMONSTRATION*



   b. By the laws of specular reflection, a parallel beam is reflected as a parallel beam from a plane mirror. DEMONSTRATION*

   c. However, if the plane mirror intercepting the parallel beam is cut into halves (or quarters), some concentration of the parallel beam is achieved by tilting each of the mirror sections "inward" DEMONSTRATION*

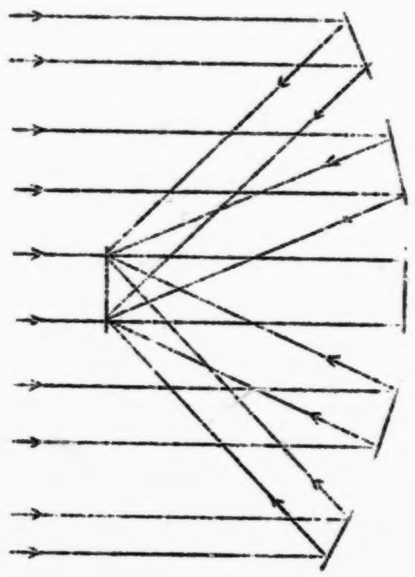


   d. The reflected parallel beams from each half (or quarter) "cross" and create a region of convergence about the size of the individual mirror sections.

   e. Where the "crossing" occurs is dependent on the relative angle between the mirror halves; it occurs closer to the mirror sections if the angle between the halves is smaller, farther away if the angle is larger.

D. Evolution of the converging mirror (continued)

3. Carrying this idea further:
   a. The region of convergence can be made smaller and smaller by using smaller and smaller mirror segments — DEMONSTRATION *
   b. The reflected beam segments can be made to converge at any chosen distance from the mirror by properly positioning each.



4. As the number of reflecting surfaces is made larger and larger the area of each mirror segment becomes smaller and smaller, so the region of convergence approaches a point (becomes "point-sized"!)
   a. If n is the number of mirror segments and A the area of each segment (also approximately the size of the region of convergence), as $n \rightarrow \infty$, $A \rightarrow 0$
   b. In the limit, the entire reflecting surface is continuous and smooth and the light is reflected to a single point; the light from the original parallel beam is brought to a FOCUS. — DEMONSTRATION *
5. The resulting surface is mathematically unique; there is only one kind of mathematical shape that can accomplish this convergence of a parallel beam to a point.
   a. This surface shape is technically referred to as a "paraboloid of revolution" (graph both branches of a parabola on sheet metal; stick the metal parabola into soft mud and spin it about the y-axis; the mud bowl thus formed is a paraboloid of revolution).
   b. More commonly this surface shape is referred to as parabolic; the mirror is called a PARABOLIC MIRROR.
   c. ONLY A CONCAVE PARABOLIC MIRROR CAN CONVERGE PARALLEL LIGHT TO A POINT.
   (d. A mathematician familiar with the geometrical properties of the parabola could predict this unique ability of the parabolic mirror to converge a beam of parallel light to a point).

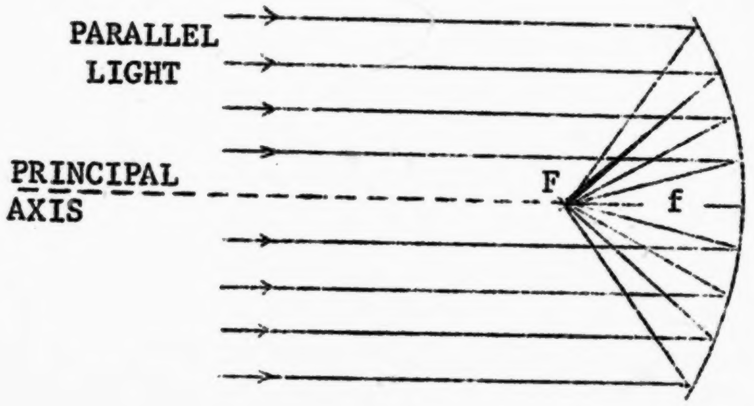
D. Evolution of the converging mirror (continued)

6. Some terms associated with converging mirrors

   a. PRINCIPAL AXIS: line drawn normal to the center of the mirror.

   b. PRINCIPAL FOCUS, F: point along the principal axis to which parallel light is converged. (This is often referred to as focal point; the use of PRINCIPAL FOCUS as the unique point for convergence of parallel light is suggested to avoid possible ambiguity).

   c. FOCAL LENGTH, f: distance from the mirror to the principal focus along the principal axis.



7. Comparison of parabolic and spherical reflecting surfaces (It is suggested that this be treated primarily --- and almost exclusively --- by way of the problem described below)

   a. In the previous unit on graphical analysis, we dealt with the parabola of form $y = k x^2$.

   b. Mathematically, a focal length f is defined which is identical to the optical focal length.

   c. In terms of this focal length, the equation of the parabola can be written $y = (1/4f) x^2$, indicating $k = (1/4f)$.

D. Evolution of the converging mirror

7. Comparison of parabolic and spherical reflecting surfaces (continued)

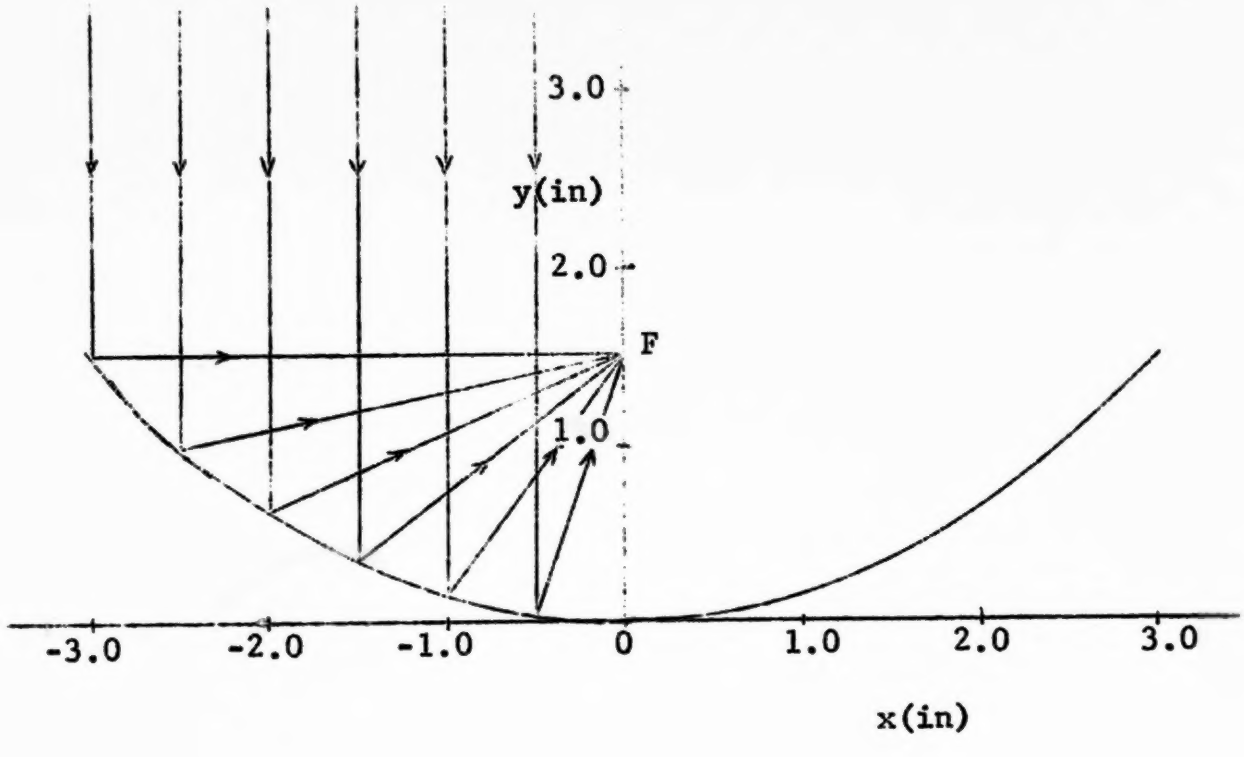
d. Draw the parabolic curve for f = 1.5 inches by graphing $y = (1/6.0)\ x^2$ for values of x from -3.0 inches to + 3.0 inches.

(1) choose the axes so that the graphed parabola is confined to the bottom quarter of the page.

(2) plot a sufficient number of points so that an accurate, smooth curve can be drawn.

(3) the given range of x values was chosen to accommodate graph paper ruled 5 squares to the inch.

e. Draw six incident rays, evenly spaced, parallel to and to one side of the principal axis; draw the reflected rays to the principal focus.

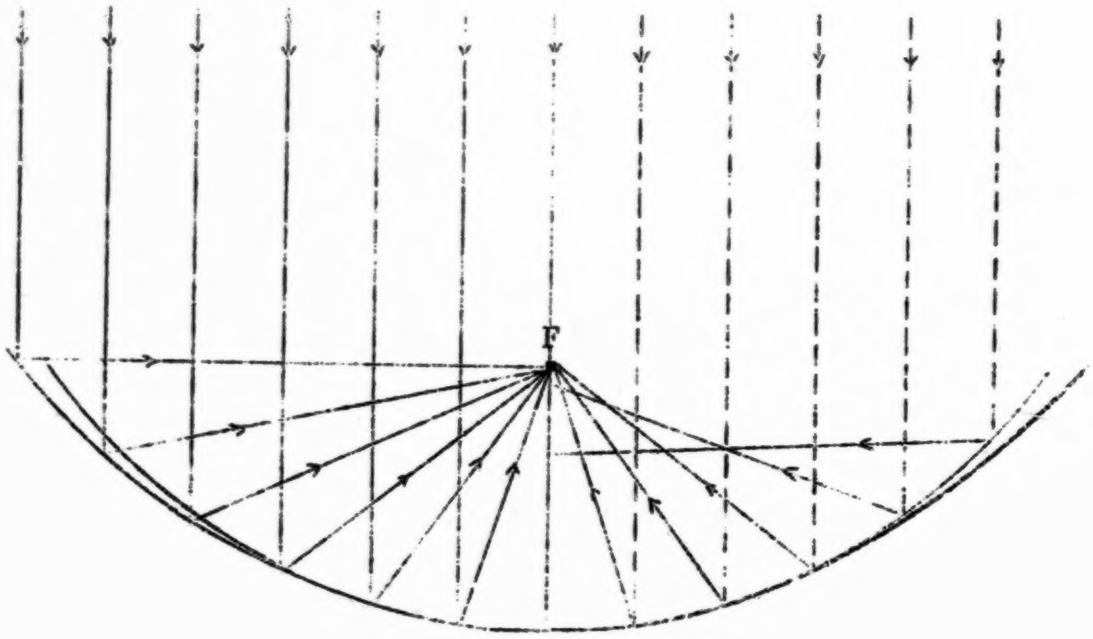


UNIT IV. REFLECTION OF LIGHT

D. Evolution of the converging mirror

7. Comparison of parabolic and spherical reflecting surfaces (continued)

   f. Draw an arc of a circle (representing a spherical surface) centered at a distance TWICE THE FOCAL LENGTH along the principal axis.



   g. Note that the two surfaces coincide in the region near the principal axis; hence the important practical conclusion that A PARABOLIC REFLECTING SURFACE OF FOCAL LENGTH f CAN BE APPROXIMATED BY A SPHERICAL REFLECTING SURFACE OF RADIUS r = 2f.

   h. It should be noted here that the extensive use of spherical surfaces to approximate parabolic ones has led to the use of the general terms CURVED, RADIUS-TYPE, CONCAVE, etc., to describe any converging mirror.

   i. Draw five rays as before, this time on the other side of the principal axis and incident on the CIRCULAR ARC.

   j. Draw (dashed lines) radii ~~to each point of~~ incidence; these radii are the NORMALS to the spherical surface at each point of incidence; (WHY?)

   k. Measure each angle of incidence and draw the corresponding reflected ray.

   l. Compare the rays reflected from this spherical surface to those reflected from the parabolic surface.

   m. Define SPHERICAL ABERRATION on the basis of this comparison.

E. Image formation by a converging mirror

1. Real image formation; description of the real image. DEMONSTRATION*

2. Laboratory investigation of the real image. EXPERIMENT *

   a. Quantities measured:

      p, distance of the object from the mirror (independent variable)

      q, distance of the image from the mirror (dependent variable)

      f, focal length of the mirror

      (if the object height h_o is recorded and the image height h_i is measured for each different object position, additional information on the MAGNIFICATION M is obtained)

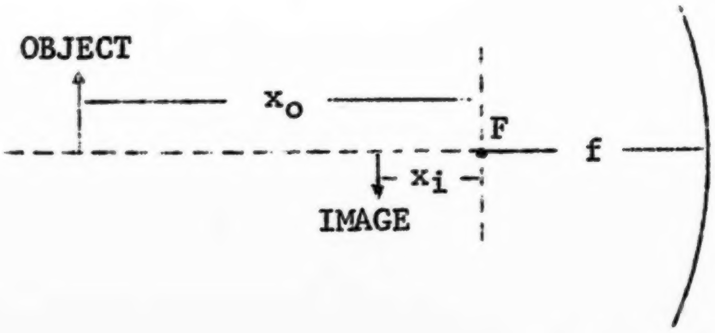
   b. While the resulting graph of q vs p resembles a hyperbola (or possibly an inverse-square curve), analysis reveals that it is not of the form

      $y = k\ (1/x)$, nor $y = k\ (1/x^2)$

3. To obtain a form more readily analyzed, define two new quantities (as Newton did), x_o and x_i.

   a. x_o is the object distance measured from the principal focus.

   b. x_i is the image distance measured from the principal focus.



   c. Since the principal focus is a distance of one focal length f from the mirror, $x_o = p - f$, and $x_i = q - f$.

REAL IMAGE FORMATION BY A PARABOLIC MIRROR.

DEMONSTRATION: (here a clear glass light bulb with coiled filament is most effective object; darkened room is also desirable if not necessary for a larger group demonstration)

* So far, the parabolic (or spherical) mirror has been used only to converge parallel light; now look at some other things it can do.

* We saw the plane mirror formed an image of an object placed in front of it; what about the image-forming possibilities of a curved mirror?

With the object (light bulb filament) placed at a distance of about three focal lengths from the mirror surface, use a piece of white paper as a screen and find the image position (just greater than one focal length distance from the mirror). For most of the students this will be the first time they have seen a real image formed, and a sharply focused, inverted real image is quite spectacular to them - use it to full advantage!

* Note that the image formed is INVERTED, SMALLER than object, and is REAL --- light must be present at image site, image is formed on the screen, or on a finger, etc.

With an image sharply in focus, keep the paper (screen) in place and move the object some distance farther from the mirror.

* Note the image falls out of focus; by now moving the paper (screen) closer to the mirror a new position is found for a sharply focused image. Repeating for several other positions (it is probably a good idea to stay well beyond 2f with the object for this part). it can be concluded that the image distance q is a function of the object distance p. where p and q are measured from the reflecting surface, along the principal axis.

* Note, too, that the image size changes with each new object position.

Now move the object toward 2f from the mirror surface; follow the image with the paper (screen).

* Note that the image increases in size and moves out away from the mirror surface.

As the object passes through 2f the image crosses from the mirror side of the object to the other side.

* Note where the image appears; note also how the image increases in size and is enlarged compared to the object.

As the object is moved in to approach a distance f from the mirror surface and the image increases in size markedly, the student is usually quite impressed if an image is formed on the opposite wall of the room. In a good-size room, a gigantic image 5' to 8' tall can be formed.

(more)

REAL IMAGE FORMATION BY A PARABOLIC MIRROR (continued)

EXPERIMENT:

* As suggested by the previous demonstration, image distance and image size vary as object distance is changed. So, measure q as a function of p (and also h_i as a function of p).

Use meter-stick optical bench; cut arrow-object from small piece of cardboard. Mount curved mirror with TACKI WAX (or similar material) on ringstand or other vertical support. Piece of white cardboard or stiff white paper can be used as a screen for locating the image.

Ideally, the object should be positioned so that the top is on the principal axis (a line perpendicular to the center of the curved surface of the mirror); then the image will appear at different distances touching and above the principal axis.

For later use in analyzing the experiment, it will be necessary to measure and record the object height, h_o, and the focal length, f, of the mirror. Measuing h_o, of course, is no problem, but the students generally will be at a loss in deciding how to measure f. Urge them to recall the definition of F, the principal focus, and its relation to f. This should suggest to them the idea of focusing the light reflected from a source at infinity, or at least a very large distance away. A lighted bulb, already set up at the far end of the room usually serves adequately as this source of parallel light. It is advisable to use a colored bulb so that the students can quickly distinguish its reflected light from the other sources in the darkened room.

In addition to the two constants (h_o and f) measured, the data consists of the measured variables p, q, and h_i. For later analysis, several other quantities will be derived from these measurements. Anticipating this, it is suggested that ten columns be arranged along the long dimension of the lab notebook as follows:

Col.1	Col.2	Col.3	Col.4	Col.5	Col.6	Col.7	Col.8	Col.9	Col.10
p(cm)			q(cm)			h_i(cm)			

Note that except for columns 1, 4, and 7, all the others are blank at this time. The students should be instructed to plot only q as a function of p at this time. The resulting graph is not one of the special cases they should recognize, even though it resembles the inverse variation curve. You may suggest they try some sample products (qp) but avoid getting involved in a complete graphical analysis of the q-p data.

(The class notes that follow introduce new derived variables x_o and x_i, as well as the linear magnification, M. In the experiment analysis, it will be convenient to use Col. 2 for x_o, Col. 3 for $1/x_o$, Col. 5 for x_i, Col. 6 for the sample products $x_i x_o$, Col. 8 for $M = h_i/h_o$, Col. 9 for $M = q/p$, and Col. 10 for computing percent differences.)

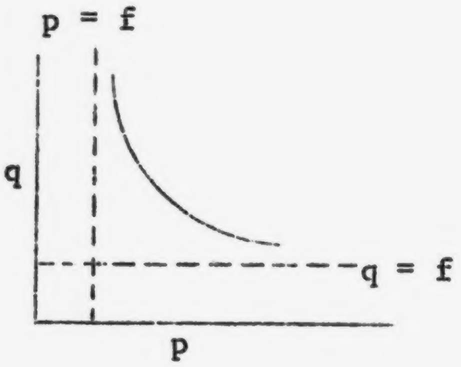
*discuss

E. Image formation by a converging mirror

3. To obtain a form more readily analyzed, define two new quantities (as Newton did), x_o and x_i. (continued)

   d. Subtract the measured focal length f from each q and p value to obtain the corresponding x_i and x_o values.

   e. Note that this transition from q vs p to x_i vs x_o can be exhibited graphically by drawing the horizontal line q = f, and the vertical line p = f (dashed lines); with these lines as axes, the contained curve is a graph of x_i vs x_o.



   f. Plot x_i vs x_o and analyze.

4. Result of analyzing x_i vs x_o graph

   a. The curve is a hyperbola of the form $y = k\,(1/x)$.

   b. The equation of the curve is $x_i = k\,(1/x_o)$, where k is a constant having units of length squared.

   c. Note that this is purely an experimental result; it is a quantitative statement of the observed behavior of the converging mirror in producing a real image.

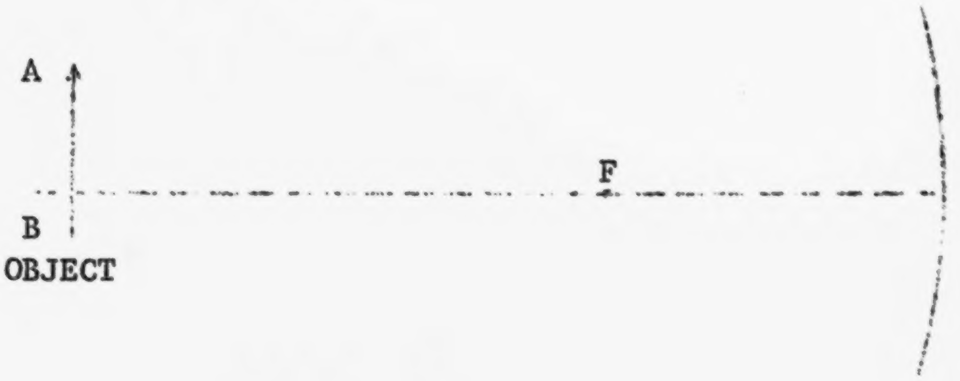
UNIT IV. REFLECTION OF LIGHT

E. Image formation by a converging mirror (continued)

5. Geometrical (theoretical) analysis

   a. Apply the rectilinear propagation of light and the laws of reflection (as they pertain to a parabolic mirror) to see how the real image is formed.

   b. Consider an object AB in front of a parabolic mirror of focal length f.



   c. Rays from each point on the object are given off in all directions; those rays that strike the mirror, upon being reflected, form the image of the object AB.

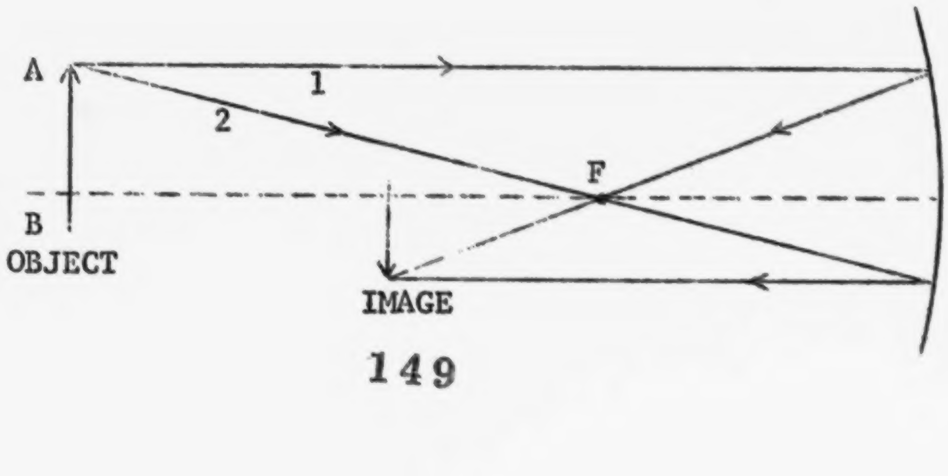
   d. The image (point-by-point reproduction of the object) is formed when rays from each object point are reconverged to form the corresponding image point (that's what image formation is all about).

   e. To find the image of any point on the object, it is necessary only to trace the path of two reflected rays: they cross at a point common to all the reflected rays originating from the same object point.

   f. To find the image of the object point A, draw the path of two principal rays:

      (1) PRINCIPAL RAY 1 is incident on the mirror in a direction parallel to the principal axis; it is therefore reflected in a direction such that it passes through F;

      (2) PRINCIPAL RAY 2 is incident on the mirror in a direction such that it passes through F; it is therefore reflected in a direction parallel to the principal axis.



E. Image formation by a converging mirror

5. Geometrical (theoretical) analysis (continued)

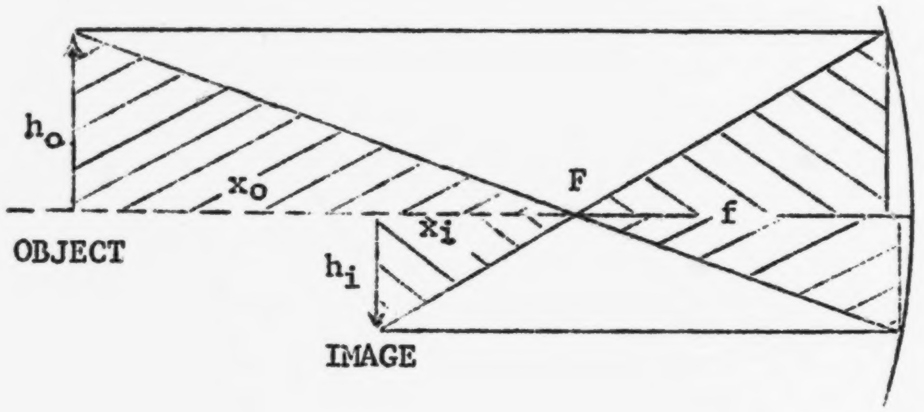
g. Reflection of PRINCIPAL RAY 2 parallel to the principal axis comes about because of the equality of the angle of incidence and the angle of reflection and points up an important property of light: THE REVERSIBILITY OF LIGHT PATHS.

h. The image of object point A occurs at the intersection of the reflected PRINCIPAL RAYS 1 and 2.

i. Repeating this same procedure for object point B, the image of this point is located and hence the whole image AB is determined.

j. Note that a ray incident on the mirror along the principal axis is reflected back along the principal axis; to simplify the geometry in what is to follow, let the tail of the object rest on the principal axis; then the tail of the image will also be on the principal axis.

k. Draw PRINCIPAL RAYS 1 and 2 as before; note that the two pairs of triangles thus formed are P-Type similar triangles.



(An approximation is involved in the triangles with vertical legs close to the mirror; the difference introduced is usually negligible except for sharply curved mirrors)

l. Since the corresponding sides of similar triangles are proportional, $h_i/h_o = x_i/f$, and $h_i/h_o = f/x_o$

m. Since x_i/f and f/x_o are both equal to the same quantity h_i/h_o, they are equal to each other: $x_i/f = f/x_o$.

n. Then $x_i x_o = f^2$ and $x_i = f^2 (1/x_o)$.

o. f is the focal length of the mirror and is therefore constant; f^2 then is also constant.

p. Compare quantitatively this theoretical result with the experimental result expressed in (4b) above.

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1. In our discussion of the geometry of image formation by a concave mirror, we arrived at the expressions $\frac{h_i}{h_o} = \frac{f}{x_o}$, $\frac{h_i}{h_o} = \frac{x_i}{f}$, and $x_i x_o = f^2$, from similar triangles.

   a. Use these expressions to predict where an object must be placed so that a real image is formed equal in size to the object.

   b. Where does this image appear?

   c. Verify by an appropriate ray diagram.

2. a. From the geometrically deduced expression $x_i x_o = f^2$ for a concave mirror, use the definitions $x_o = p - f$ and $x_i = q - f$ to arrive at the MIRROR EQUATION, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$.

   b. IN LAB BOOK, plot a graph of $\frac{1}{q}$ vs $\frac{1}{p}$ for the data obtained with the parabolic mirror; interpret in the light of the Mirror Equation.

   c. Does the Mirror Equation apply to a plane mirror? Comment quantitatively.

3. LINEAR MAGNIFICATION M is defined as the ratio of the image size to the object size (along a single dimension), or h_i/h_o. GEOMETRICALLY, find a simple expression for the linear magnification M in terms of the (absolute values of) q and p.

4. A concave mirror is cut from a sphere of radius 18 centimeters. Draw full scale ray diagrams showing the formation of the image of a vertical object 2.0 centimeters in height placed at a distance from the mirror of

   a. 24 cms
   b. 21 cms
   c. 18 cms
   d. 15 cms
   e. 12 cms
   f. 9 cms
   g. 4.5 cms
   h. 3 cms

   WORK THIS PROBLEM ON A SEPARATE SHEET OF EXTRA LENGTH

5. Apply the Mirror Equation to each part of problem 4 (above) to find the image distance and the image height. Compare to measured values obtained from the ray diagrams.

UNIT IV. REFLECTION OF LIGHT

F. The MIRROR EQUATION

1. The result obtained on the previous page, $x_i = f^2 (1/x_o)$, or $x_i x_o = f^2$, is called the NEWTONIAN FORM of the MIRROR EQUATION.
    - a. The quantities x_o and x_i are not always conveniently known or directly obtainable.
    - b. An expression describing the mirror involving p and q instead may be more desirable.
2. Since $x_i = q - f$, and $x_o = p - f$, substitute these expressions for x_i and x_o in the equation $x_i x_o = f^2$.
    - a. Then $(q - f)(p - f) = f^2$ and $qp - qf - fp + f^2 = f^2$ or $qp - qf - fp = 0$.
    - b. Dividing by the product qpf: $qp/qpf - qf/qpf - fp/qpf = 0$ and $1/f - 1/p - 1/q = 0$.
    - c. Rearranging, $1/p + 1/q = 1/f$.
3. $1/p + 1/q = 1/f$ is called the GAUSSIAN FORM of the MIRROR EQUATION. (this form is far better known than the Newtonian $x_i x_o = f^2$, and is more readily remembered and convenient to use)
4. While our derivation of the MIRROR EQUATION was based on the real image formed by a concave, parabolic mirror, $1/p + 1/q = 1/f$ is also valid for the formation of virtual images with both concave and convex mirrors; the following rules apply:
    - a. If the image is REAL, the value of q is positive.
    - b. If the image is VIRTUAL, the value of q is negative.
    - c. The focal length f is positive for CONCAVE mirrors.
    - d. The focal length f is negative for CONVEX mirrors.
5. The MIRROR EQUATION and the plane mirror
    - a. Imagine constructing parabolic (or spherical) mirrors of longer and longer focal length.
    - b. As the focal length is made increasingly longer, the surface becomes less sharply curved and the radius of curvature becomes greater and greater.
    - c. As the radius of curvature increases without limit, the mirror approaches a plane surface, and the focal length approaches infinity.
    - d. Since the focal length appears in the denominator of the MIRROR EQUATION, $1/p + 1/q = 1/f$,

$$\text{as } f \longrightarrow \infty \text{ , } 1/f \longrightarrow 0$$

    - e. Under this condition, the MIRROR EQUATION becomes $1/p + 1/q = 0$; then $1/p = -1/q$, and $q = -p$, which is exactly the result previously observed for the plane mirror.
    - f. Hence, with this interpretation, the MIRROR EQUATION is also applicable to the formation of the virtual image by a plane mirror.

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1. The sun is at a distance of approximately 1.5 X 10^{11} meters from the earth, and has a diameter of about 1.4 X 10^{9} meters. If the HALE REFLECTING TELESCOPE at the PALOMAR OBSERVATORY has a focal length of 18 meters:

   a. How large an image of the sun is formed by this parabolic mirror?

   b. At what distance from the mirror is this image formed?

2. A concave mirror has a focal length of 20 cms.

   a. Where should an object be placed if the image distance is to be three times the object distance?

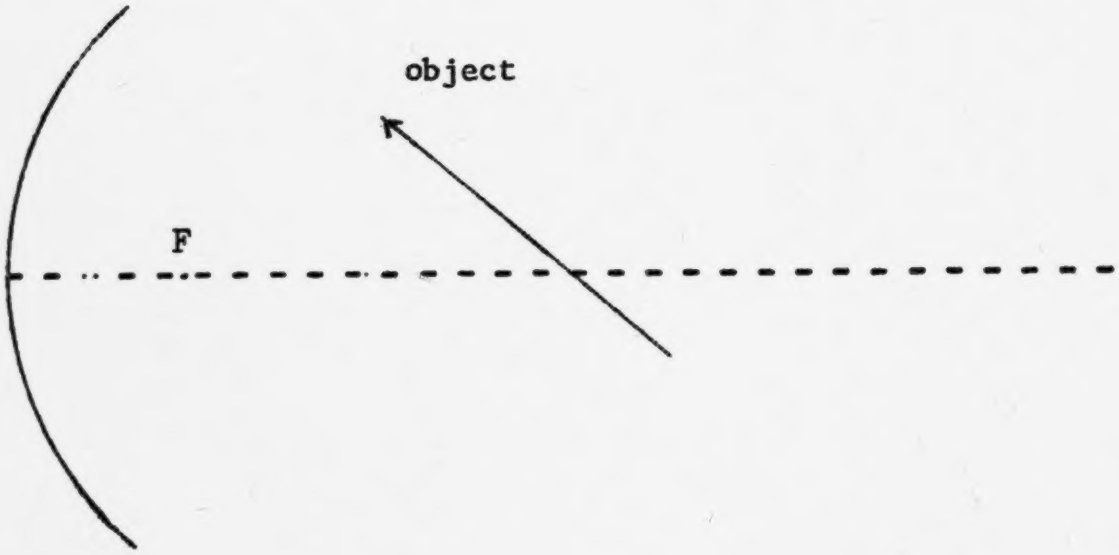
   b. If the object is placed at a distance of 10 cms. from the mirror, calculate the image distance and the linear magnification.

3. An ornamental silvered sphere has a diameter of 2.0 feet.

   a. When a six-inch fountain pen is placed 15 inches from the mirror, where is the image formed?

   b. What is the image height?

4. Draw the appropriate ray diagram showing the formation of the image:



G. Formation of a virtual image by a parabolic concave mirror

1. It was observed that no real image is formed for object positions inside the principal focus: no real image for $p < f$.

2. However, an erect VIRTUAL image is observed at a location apparently behind the mirror for object distances less than the focal length. DEMONSTRATION

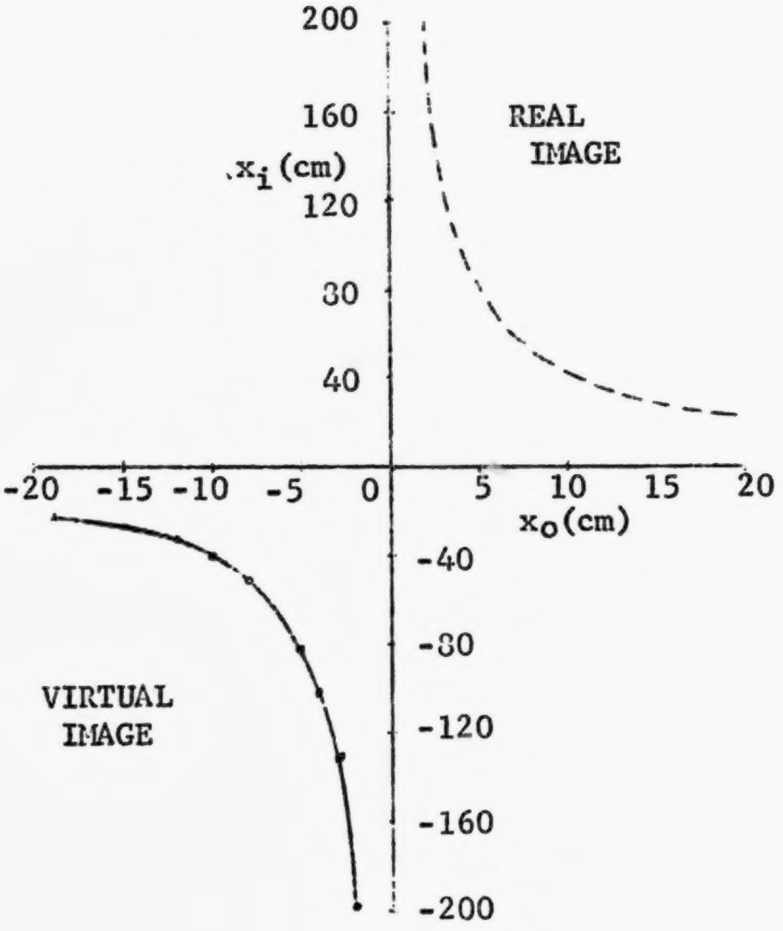
   a. The image size increases as the object is moved from the mirror toward F.

   b. The apparent position of the image moves farther behind the mirror as the object is moved from the mirror toward F.

3. Laboratory investigation of the virtual image. EXPERIMENT **

   a. Both x_o and x_i are negative for the virtual image.

   b. x_i vs x_o plot gives a hyperbola in the third quadrant of the form $y = k\ (1/x)$; the graph drawn below is for a mirror of focal length $f = 20.0$ cm.



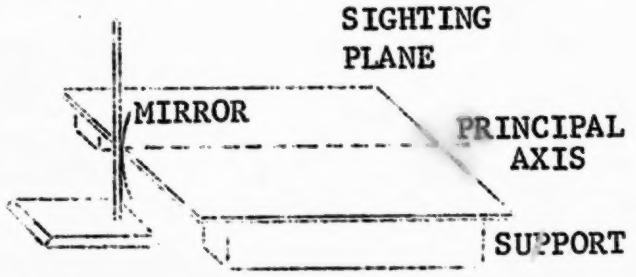
   c. Note that the data for the virtual image gives a curve symmetrical to the real image result (dashed curve), i.e., the second branch of the hyperbola $x_i = k\ (1/x_o)$

   (d. Of the curves studied here in graphical analysis, the hyperbola, parabola, and inverse square all mathematically have second branches; it is informative and sometimes fruitful to investigate the PHYSICAL SIGNIFICANCE of the mathematically predicted branch)

NAME______________________

LABORATORY INSTRUCTION SHEET: Virtual Image by a Concave Mirror

1. Measure and record the focal length of the mirror. (Ideally, the mirror should be the same one used in investigating the real image): f = ______cm.
2. Mount the mirror on the ringstand (using TACKIWAX or EDMUND-TAK) so that the "sighting plane" cardboard is aligned along the principal axis of the mirror.



3. Draw a line across the center of a sheet of paper; position the paper on the cardboard so that the line coincides with the principal axis of the mirror.
4. Mark off 5 different values of p less than the focal length along the axis, taking care that the measurements are made from the mirror surface.
5. Take three sightings at the image of an object pin placed at each of these p values; mark each sighting line carefully.

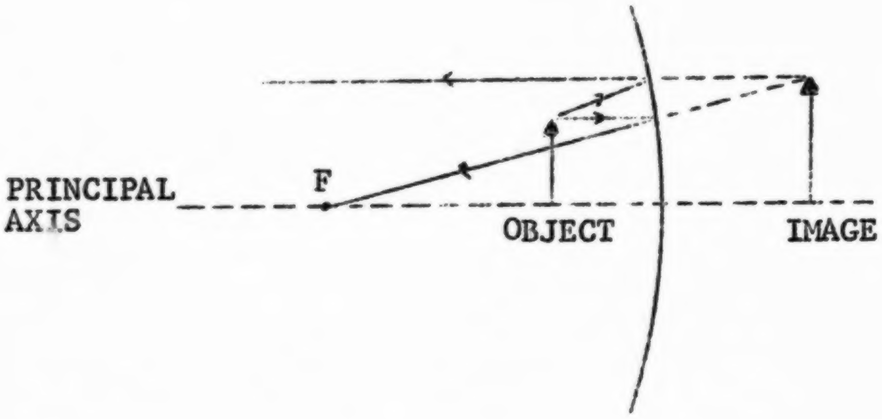
p (cm)	x_i (cm)	q (cm)	x_i (cm)			

6. Locate each image position (value of q) by extending the sighting lines behind the mirror; record these q values as negative since they represent the positions of the virtual image behind the mirror.
7. Calculate x_o and x_i values corresponding to each p and q.
8. Plot x_i vs x_o.
9. Analyze the x_i vs x_o graph; compare and relate this result to that obtained for the real image with the same mirror.
10. For the p values above, find the q values predicted by the MIRROR EQUATION, $1/p + 1/q = 1/f$; record these values in one of the blank columns above; compare these values with those obtained experimentally.

G. Formation of a virtual image by a parabolic concave mirror (continued)

4. Ray diagram for virtual image formation

   a. Place an object in front of the concave mirror at a distance p less than the focal length f.



   b. Draw PRINCIPAL RAY 1 from the top of the object to the mirror; this ray incident on the mirror parallel to the principal axis is reflected in a direction through the principal focus F.

   c. Draw PRINCIPAL RAY 2 from the top of the object to the mirror; this ray is incident on the mirror in the direction of a line that passes through F and hence is reflected parallel to the principal axis.

   d. These diverging reflected rays give the appearance of light originating at a single point behind the mirror; this is exactly what constitutes a virtual image.

   e. Extending the diverging reflected rays backward (dashed lines) gives the location of the virtual image.

   f. To compare the theoretically predicted image distance with those observed experimentally, draw the appropriate rays diagrams for each object distance investigated.

5. The virtual image and the MIRROR EQUATION

   a. Both the experimental (see 3 above) and the theoretical (see 4 above) results confirm the validity of applying the MIRROR EQUATION to virtual image formation.

   b. Write the MIRROR EQUATION, $1/q = 1/f - 1/p$.

   c. Since $p < f$ for the virtual image, $1/p > 1/f$, and the negative term on the right side predominates.

   d. Hence, $1/q$ is negative, and all q values will be negative as well for the virtual image.

H. Formation of an image by a CONVEX mirror

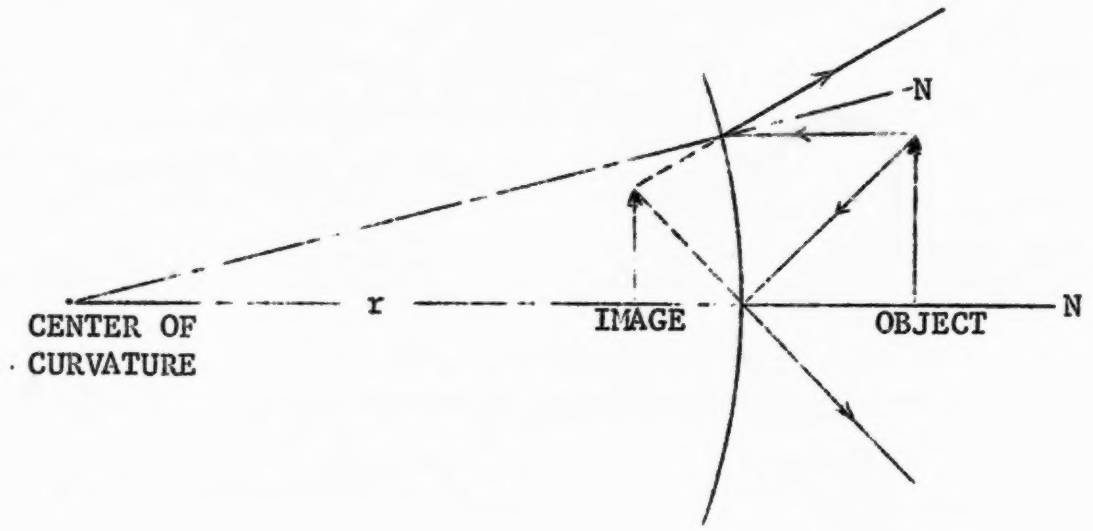
1. Observations

DEMONSTRATION

   a. A convex surface does not converge parallel light.
   b. Light diverges from a convex surface: DIVERGING MIRROR.
   c. Regardless of where the object is placed, NO REAL IMAGE IS FORMED BY A CONVEX MIRROR.
   d. For every object position a VIRTUAL IMAGE is formed by the convex mirror.
   e. The image moves farther behind the mirror as the object is moved farther from the mirror surface.

2. Ray diagram

   a. Place an object (at any distance) in front of a convex mirror whose surface curvature is given by the radius r.



   b. Mark off on the concave side of the curved surface a distance equal to the radius of curvature r; RADII DRAWN FROM THIS POINT GIVE THE DIRECTIONS OF THE NORMALS TO THE REFLECTING CONVEX SURFACE.
   c. Draw ANY two rays incident on the mirror surface from the top of the object; measure the angle of incidence and draw the reflected ray at an angle of reflection equal to the angle of incidence.
   d. These reflected rays are diverging and appear to originate at a single point behind the mirror.
   e. Extending the reflected rays behind the mirror gives the location of the virtual image.

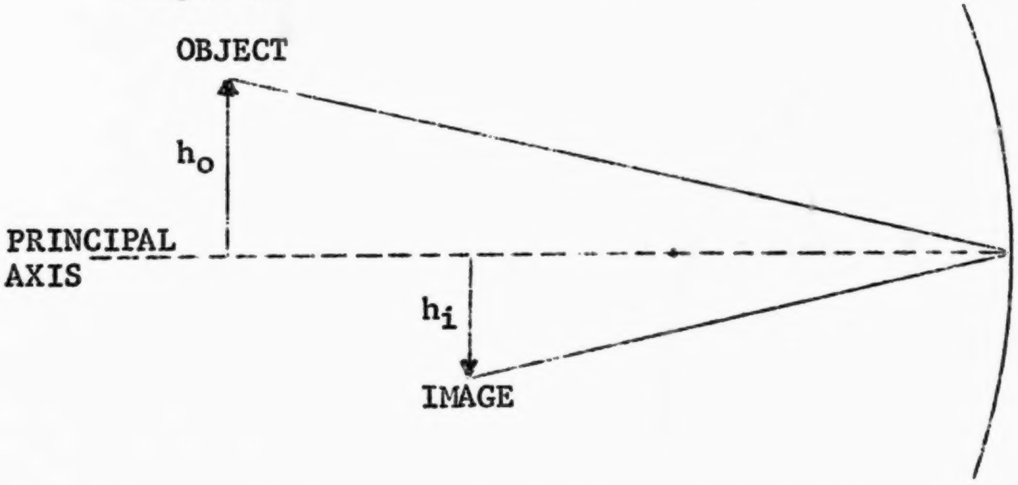
3. The MIRROR EQUATION applies if a quantity "f" = $\frac{1}{2}r$ is defined to replace the lacking focal length f.

I. Linear Magnification

1. The LINEAR MAGNIFICATION M is defined as the ratio of the image size to the object size along a single dimension: $M = h_i/h_o$.

2. An expression for M in terms of p and q can be obtained as follows:

   a. For an object placed at some distance p from the mirror, locate the image by drawing PRINCIPAL RAYS 1 and 2 (not shown in the diagram).



   b. Since all the rays leaving a given object point and incident on the mirror are reconverged to the same image point, a ray leaving the top of the object incident where the principal axis intersects the mirror, is reflected to this same point on the image.

   c. Note that S-Type Similar Trinagles are formed, since the angles formed by the incident and reflected rays are equal (the principal axis is normal to the surface).

   d. Since the horizontal legs of the triangles are p and q, respectively, $h_i/h_o = q/p$.

   e. But, by definition h_i/h_o is M, so $M = q/p$.

3. While $M = q/p$ has been obtained by considering a real image formed by a concave mirror, the absolute value of the ratio q/p gives the magnification for both real and virtual images formed by any type (concave, convex, plane) of mirror: $M = |q/p|$

4. For a PLANE mirror, since $q = -p$, $M = 1$ (this is consistent with the experimentally observed $h_i = h_o$, which also gives $M = 1$ for this virtual image).

5. For a CONCAVE mirror (these deductions can be made quantitatively by reference to the MIRROR EQUATION):

   a. if $p < f$, $M > 1$: the virtual image is LARGER than the object.

   b. if $f < p < 2f$, $M > 1$; the real image is LARGER than the object.

   c. if $p = 2f$, $M = 1$; the real image is the SAME SIZE as the object.

   d. if $p > 2f$, $M < 1$; the real image is SMALLER than the object.

   e. How about $p = f$? NO image because the light reflected by the mirror is parallel and does not converge to form an image.

6. For a CONVEX mirror: for any p, $M < 1$; the virtual image is always SMALLER than the object (except in the trivial case at $p = 0$, $M = 1$).

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Select by letter the ITEM from the LIST that best fits the STATEMENT:

LIST

A. 5.0
B. 2.5
C. -2.5
D. 1.5
E. -1.5
F. 1.0
G. -1.0
H. magnification
I. plane mirror
J. concave mirror
K. convex mirror
L. focal point
M. $1/p + 1/q = 1/f$
N. virtual image
O. real image
P. $x_i = q - f$
Q. $x_o = p - f$
R. principal axis
S. principal focus
T. $x_i x_o = f^2$
U. focal length
V. normal

STATEMENT

1. Ratio of the image size to the object size along a single dimension 1________

2. Newtonian Form of the Mirror Equation 2________

3. The focal length of a parabola described by the expression $y = 0.10x^2$, all values in cms. 3________

4. Image formed by the actual intersection of reflected light rays. 4________

5. Curved specular surface capable of forming only a virtual image. 5________

6. Constructed line perpendicular to a surface at the point the incident ray strikes. 6________

7. A parabolic surface of focal length 2.5 cms can be approximated by a spherical surface who radius is (in cm) 7________

8. Point at which a parabolic mirror reflects an incident beam of parallel light. 8________

9. Image which the eye sees by following diverging rays back to their apparent origin. 9________

10. Specular surface for which the image and object distances are always equal. 10________

Choose by letter those features that describe the resulting image in each case; you may find a rough scale diagram helpful.

A. Virtual
B. Real
C. No Image
D. SMALLER than object
E. SAME SIZE as object
F. LARGER than object

11. An object is placed in front of a plane mirror. 11________

12. An object is placed at a distance of 30 cms in front of a concave mirror of radius 40 cms. 12________

13. An object is placed at a distance of 30 cms in front of a concave mirror whose focal length is 10 cms. 13________

14. An object is placed at a distance of 10 cms in front of a concave mirror whose radius is 20 cms. 14________

15. An object is placed at a distance of 10 cms in front of a concave mirror cut from a sphere whose diameter is 80 cms. 15________

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16. A girl looks into a mirror from a distance of 25 cms and sees her face enlarged by a factor of two.

    a. What kind of a mirror is it (plane, concave, convex)? Why?

    16a____________________

    b. Find the focal length of the mirror.

    16b____________________

17. A fly "buzzes" an ornamental silvered sphere whose diameter is 1.0 ft. If the "wing span" of the fly is ½ inch and the fly is at a distance of 3.0 inches from the mirror,

    a. What is the apparent position of the fly's image?

    18a____________________

    b. How large is the wing span of the fly's image?

    18b____________________

18. Show GEOMETRICALLY how we arrived at the expression $x_i x_o = f^2$.

UNIT V. REFRACTION OF LIGHT

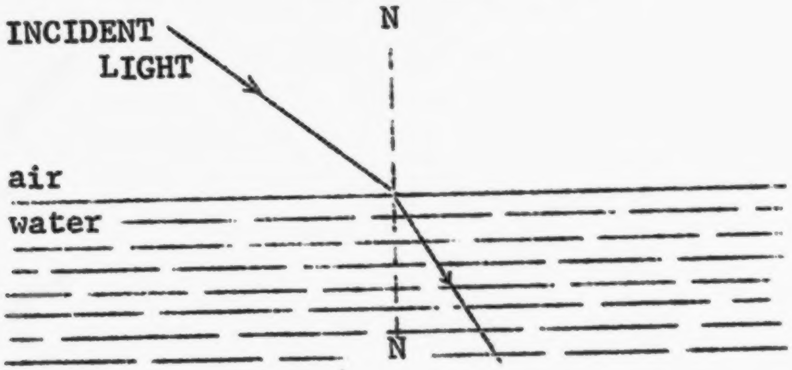
DEMONSTRATION......

A. Observations and definition of refraction

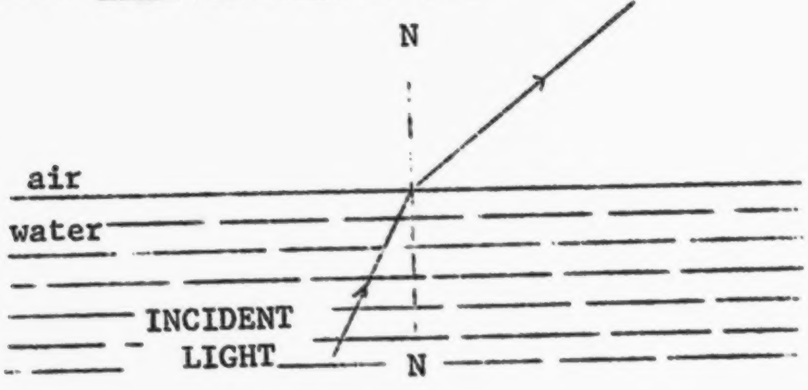
1. "Coin in cup" --- adding water to a cup with a coin at the bottom makes the coin appear to "float" near the water surface.

2. Straight stick in water --- stick appears to "crack" at water surface (a variation is also effective: pre-bend by heating a plastic ruler; position it with the "crack" at the water surface so that the bent ruler appears straight!)

3. Pencil of light from air to water (a collimated flashlight beam and a transparent rectangular tank are all that is necessary; a small amount of FLUORESCEIN dye renders the light path in water clearly visible).
   a. Light incident along the normal is not changed in direction as it enters the water from air.
   b. Light incident at other angles abruptly changes direction at the water surface; the new direction in water is inclined toward the normal.



4. Pencil of light emerging from water into air (supporting the tank on bricks allows the flashlight to be placed underneath).
   a. Light incident along the normal is not deviated from a straight-line path as it enters the air from water.
   b. Light incident at other angles abruptly changes direction as it emerges into air; the new direction in air is inclined away from the normal.



A. Observations and definition of refraction (continued)

5. Discussion and conclusions

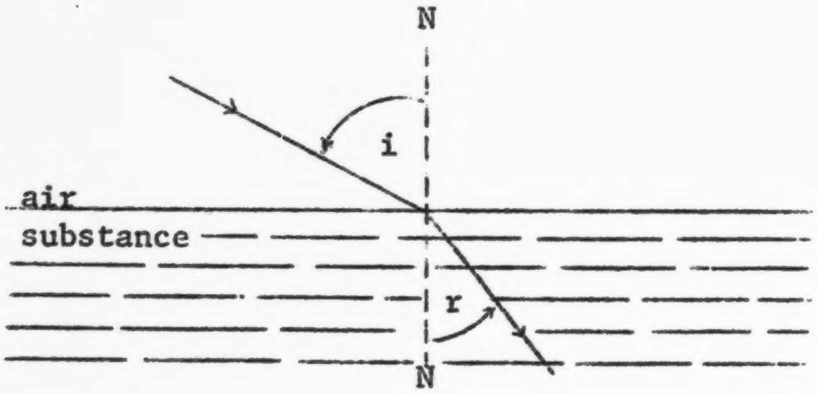
   a. The DIRECTION of the light path is changed; the path in each substance is a straight line.

   b. The change occurs at the boundary or INTERFACE between transparent media.

   c. The change in direction takes place sharply; it is not a gradual curving of the path.

   d. Define REFRACTION: THE ABRUPT CHANGE IN DIRECTION OF LIGHT AT THE INTERFACE BETWEEN TWO DISSIMILAR TRANSPARENT MEDIA.

   e. To describe refraction quantitatively, use the ANGLE OF INCIDENCE i and the ANGLE OF REFRACTION r (note that both i and r are measured from the normal)



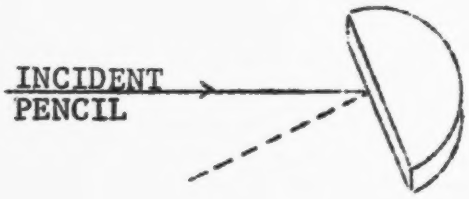
   f. Note that the angle of refraction depends on the angle of incidence: $r = f(i)$.

   g. For light from air to water, $i > r$; for light from water to air, $i < r$.

B. Quantitative observations

1. Optical Disk DEMONSTRATION

   a. Pencil of light in a fixed direction is incident at the center of the plane edge of a semicircular glass (or plastic) slab.

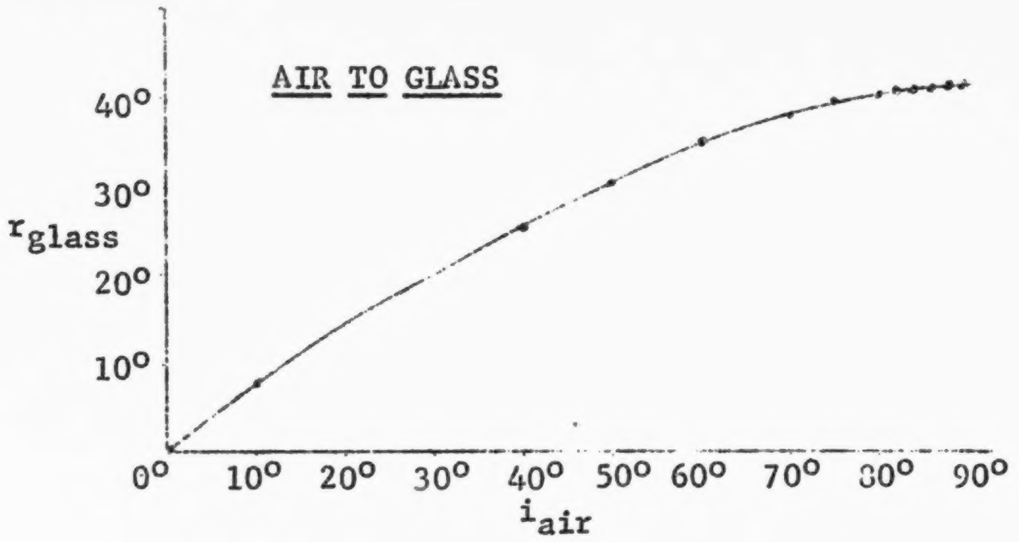


   b. The transparent slab is rotated to vary the angle of incidence.

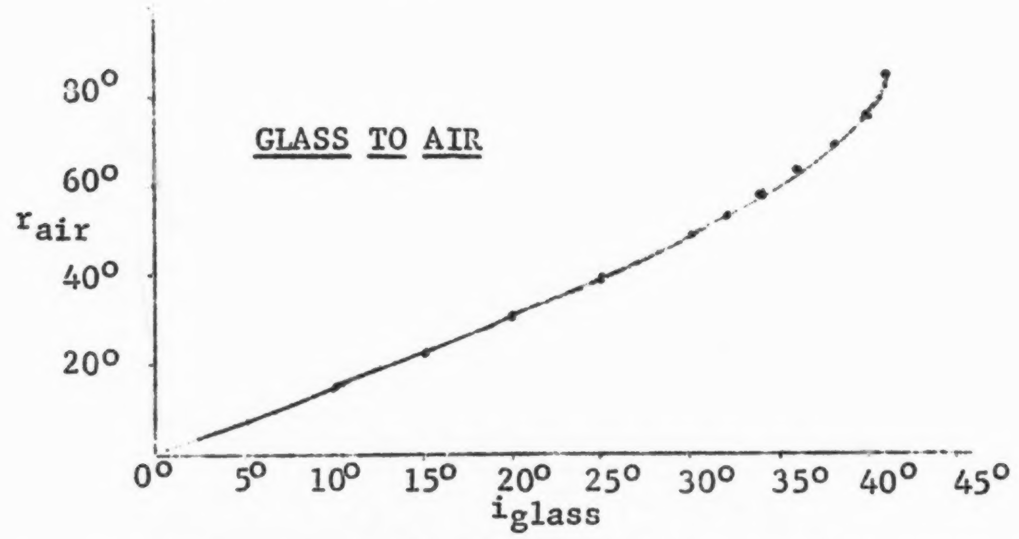
UNIT V. REFRACTION OF LIGHT

B. Quantitative Observations

1. Optical Disk (continued)
    - c. Refraction occurs as the light enters the glass from air; the refracted light in glass emerges into air without again changing direction --- WHY?
    - d. The optical disk permits accurate determinations of the incident and refracted angles to 0.5 degrees.
2. Data for light from air to glass: r vs i
    - a. Plot r_{glass} vs i_{air}



    - b. Resulting graph is a curve; of the special cases studied, it most resembles the square-root curve.
    - c. A quick check indicates i and r are not related by the square root; look for something else.
3. Data for light from glass to air: r vs i.
    - a. Light incident on the curved edge of the glass slab and directed radially inward is refracted only in emerging from glass into air.
    - b. Plot r_{air} vs i_{glass}



    - c. Resulting graph is a curve; of the special cases studied, it most resembles a parabola, or variation with the square.
    - d. A quick mathematical check indicates i and r are not related by the square; look for something else.

UNIT V. REFRACTION OF LIGHT

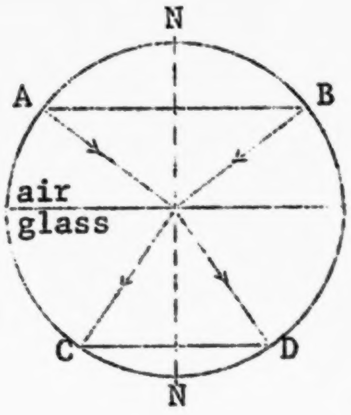
C. Something else ... Snell Circles and Snell's Law

1. Willebrord Snell (1621); the geometry of refraction EXPERIMENT **

2. Draw incident and refracted rays at the measured angles on a circle whose bottom half represents the semicircular glass slab.
   - a. See the prepared Snell Circle sheet.
   - b. Even though measurements were made for incident rays to one side of the normal only, it is reasonable to expect the same incident-refracted pairs to the other side of the normal as well.
   - c. Drawing these symmetrical incident and refracted rays will assist in the measurements to follow.

3. Draw the CHORDS AB (in air) for each incident direction.
   - a. Note that the chords are drawn perpendicular to the normal.
   - b. Snell and others to follow used semichords only for the same type of analysis.

4. Draw the CHORDS CD (in glass) for each refracted direction; these chords are also perpendicular to the normal (and parallel to chords AB)



5. On a second prepared Snell Circle, repeat the procedure for the glass-to-air path.

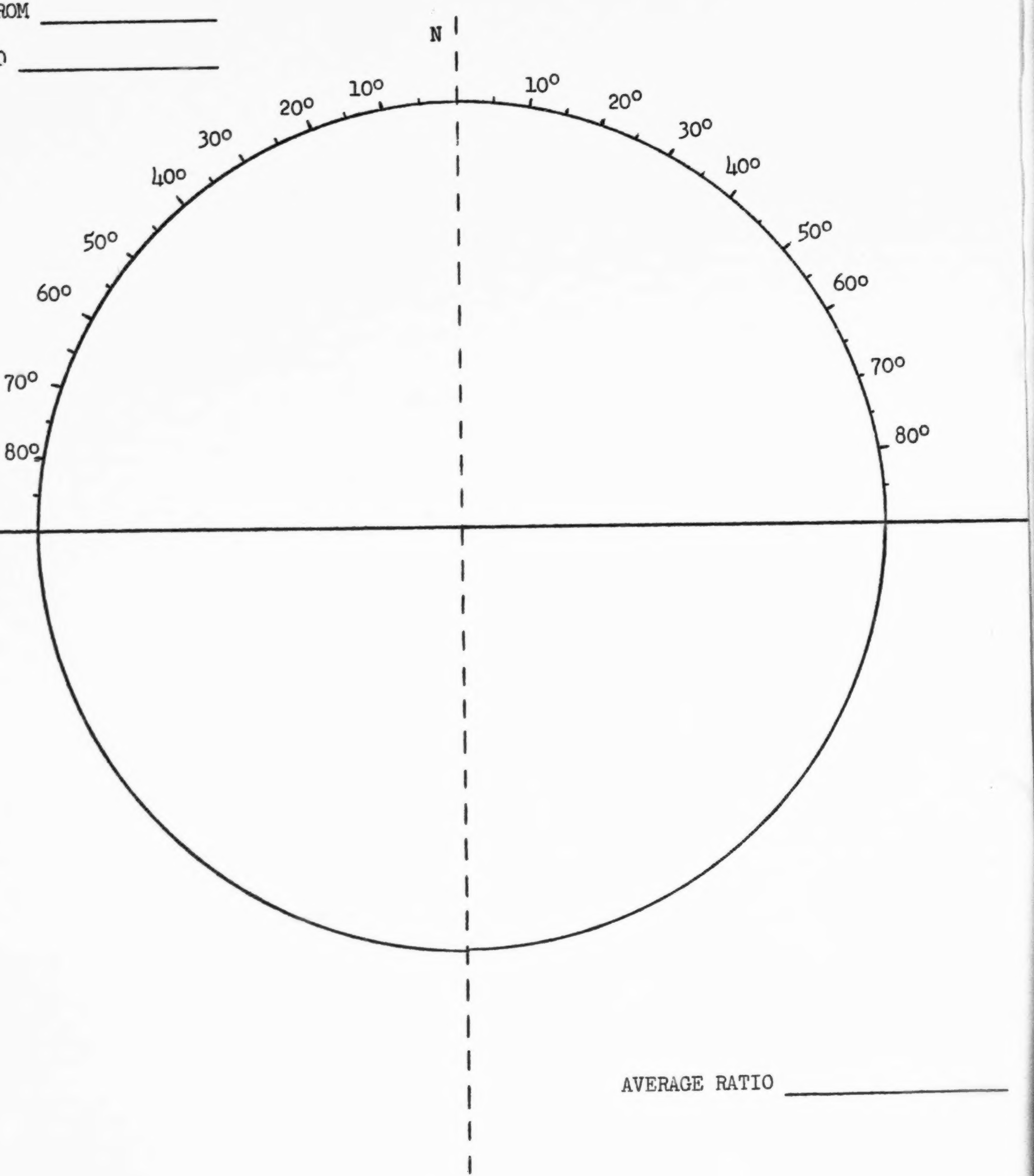
6. For both paths, air to glass and glass to air,
   - a. Measure the length of each incident chord.
   - b. Measure the length of each refracted chord.
   - c. Plot the INCIDENT vs REFRACTED CHORDS for each path.

QUANTITATIVE SCIENCE

NAME ______________________________

FROM ______________

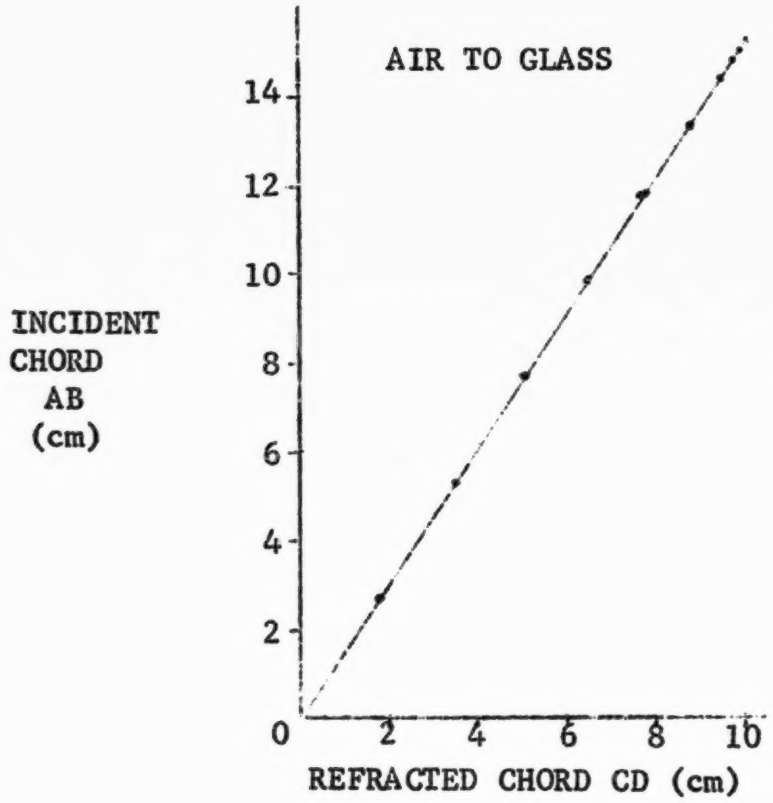
TO ______________



AVERAGE RATIO ________________

C. Something else ... Snell Circles and Snell's Law (continued)

7. For light from air to glass:



a. Result of plotting AB (incident chord) vs CD (refracted chord) is a STRAIGHT LINE THROUGH THE ORIGIN.

b. Hence, AB = (slope) CD.

c. Since the slope of a line is constant, the result can also be expressed, AB/CD = constant, for light from air to a given substance.

d. This dimensionless constant is called the INDEX OF REFRACTION of the substance (glass, in this case), and has the symbol n.

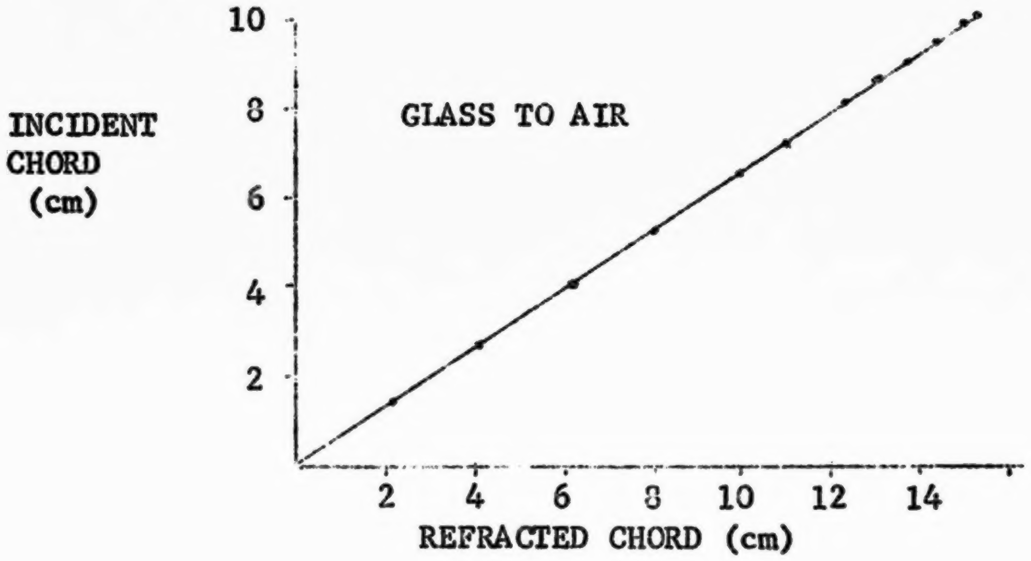
e. Hence, AB/CD = n, or

$$\frac{\text{CHORD IN AIR}}{\text{CHORD IN SUBSTANCE}} = \text{INDEX OF REFRACTION OF SUBSTANCE}$$

f. This conclusion is known as SNELL'S LAW.

C. Something else ... Snell Circles and Snell's Law (continued

8. For light from glass to air:



a. Again the result is a STRAIGHT LINE THROUGH THE ORIGIN but of different slope than for the air-to-glass path.

b. The slope of this line is seen to be the reciprocal of the slope found above; this slope, then, is 1/n, or the reciprocal of the index of refraction of glass.

c. This indicates that the path of light from glass to air is just the reverse of the light path from air to glass; more evidence for the REVERSIBILITY OF LIGHT PATHS.

d. The REVERSIBILITY OF LIGHT PATHS is also shown analytically by the following:

(1) interchange the roles of the angles of incidence and refraction in the original r vs i data for the glass-to-air path.

(2) superimpose these switched values on the r vs i graph for the air-to-glass path.

(3) all points will lie on the same curve; conclude.

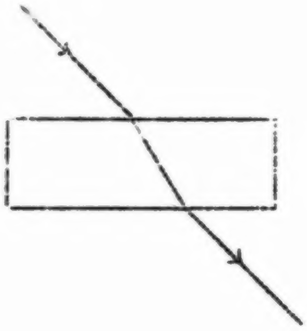
UNIT V. REFRACTION OF LIGHT (continued)

D. Laboratory: the index of refraction of liquids — EXPERIMENT

1. The method is similar to that described in the PSSC PHYSICS LABORATORY GUIDE, with the following exceptions:
   a. The semicircular, transparent box is placed on a pre-drawn Snell Circle.
   b. The incident and refracted chords are drawn on the circle and measured.
   c. A graph is made of the incident and refracted chords for both paths.
   d. The index of refraction is found from the slopes of the lines.
2. After refraction in water is investigated, the same procedure and materials can be used to study and compare refraction in ice.
3. Several transparent liquids may be made available as "unknowns".
   a. The measured n from the slope may be checked against Handbook values of possible liquids.
   b. Some suggested liquids: corn oil, light tea, castor oil, clear Karo, olive oil, kerosene, mineral oil, glycerol, ethylene glycol, Vodka, etc.

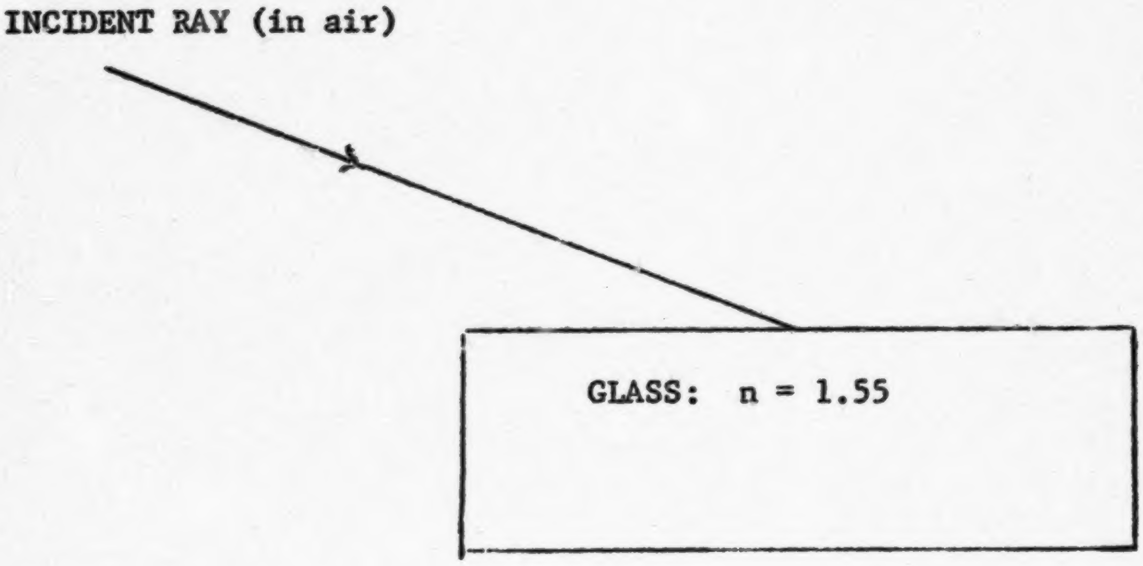
E. Convergence of a parallel beam of light by refraction

1. A pencil of light incident on a transparent material with parallel sides emerges in a direction parallel to the original direction, regardless of the direction of incidence. (see Exercise sheets) — DEMONSTRATION



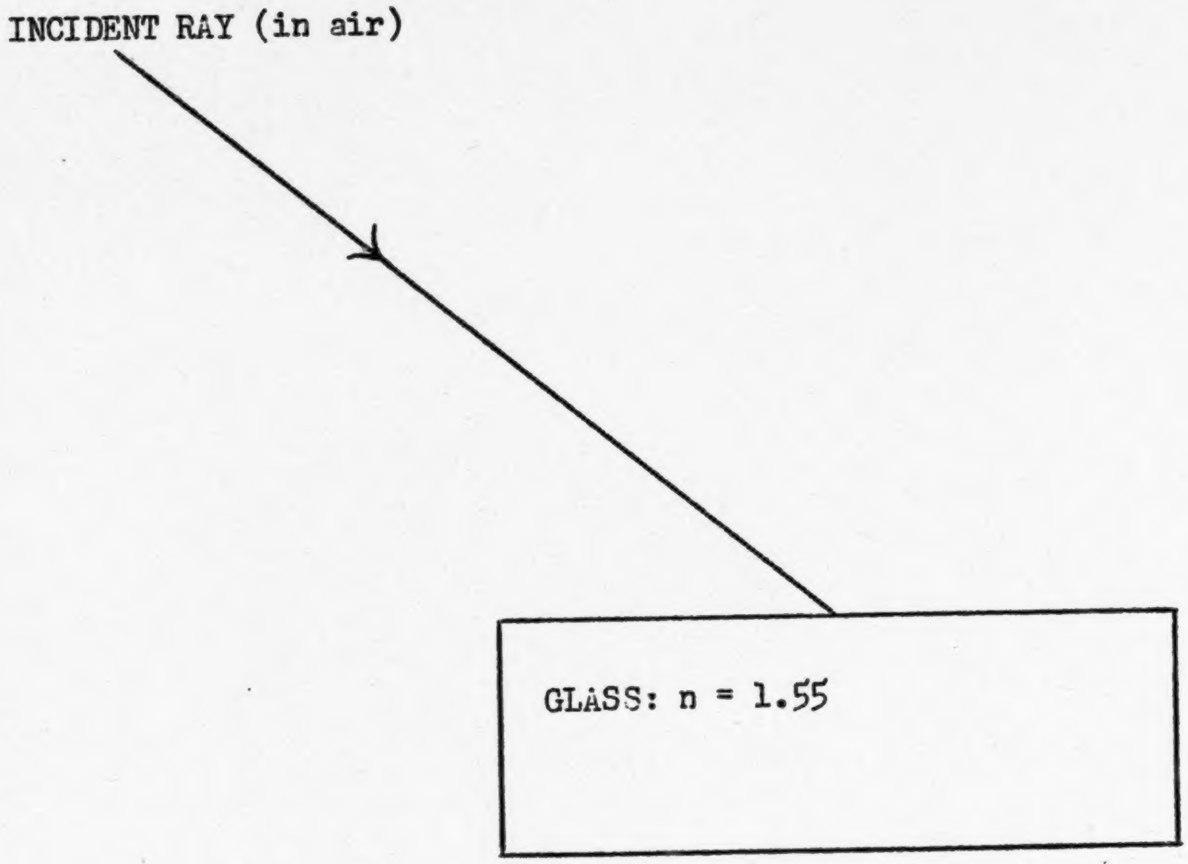
QUANTITATIVE SCIENCE DATE________________ NAME______________________

Light is incident on a block of glass as shown. Use Snell's Law to determine the path of light through the glass and then emerging into air. Conclude.



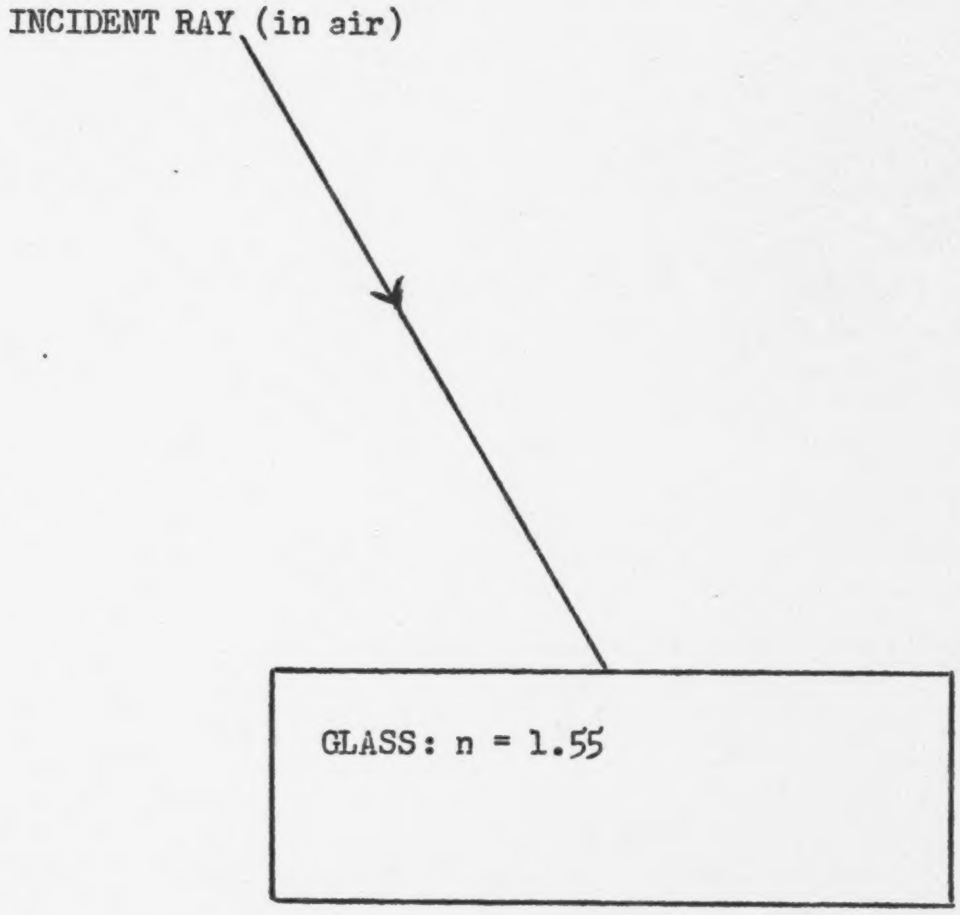
QUANTITATIVE SCIENCE DATE__________ NAME____________________

Light is incident on a block of glass as shown. Use Snell's Law to determine the path of light through the glass and then emerging into air. Conclude.



QUANTITATIVE SCIENCE DATE____________ NAME____________________

Light is incident on a block of glass as shown. Use Snell's Law to determine the path of light through the glass and then emerging into air. Conclude.

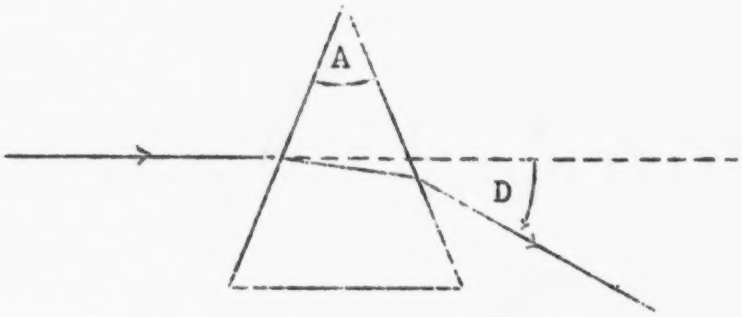


E. Convergence of a parallel beam of light by refraction (continued)

2. A pencil of light incident on a prism is changed in direction.

   a. An optical prism is a transparent object whose sides are not parallel. DEMONSTRATION

   b. The refracting ability of a prism is determined by two factors: its shape (given by the APEX ANGLE A) and its INDEX OF REFRACTION n.



   c. The total departure of the light path from its original direction is measured by the ANGLE OF DEVIATION D.

   d. Note that two refractions occur: at the air-prism interface and again at the prism-air interface.

   (e. It may be worthwhile to emphasize that it is only the change in direction that is of concern here; to most students a prism suggests color and invariably the question is raised).

3. To determine the effect of the size of the apex angle A on the deviation by a prism of a given index of refraction, EXPERIMENT *

   a. Draw a ray incident on a prism of a given A in a direction parallel to the base.

   b. Use Snell's Law to predict the path of the ray through and out of the prism.

   c. Compare the deviations produced by prisms of the same n and different A.

EVOLUTION OF A CONVERGING LENS — INSTRUCTOR INFORMATION SHEET

(1) It is suggested that this exercise be done by students in groups of three.

(2) Distribute the prepared sheets on which is drawn a prism of a given n and A.

(3) Each group member receives two sheets with prisms of the same A.

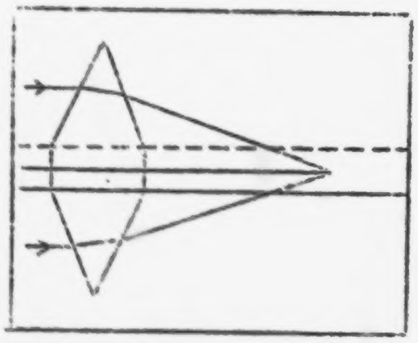
(4) The prisms are of three different apex angles, $A = 35^\circ$, $A = 45^\circ$, and $A = 55^\circ$; each student in a group receives prisms of apex angle different from the others in the same group.

(5) Drawing a Snell Circle at the point of incidence permits an immediate determination of the incident chord; dividing this measured value by the given index of refraction n predicts the size of the refracted chord; the refracted chord is then drawn in on the Snell Circle and the ray's path in the prism is determined.

(6) Drawing a second Snell Circle centered at the point of emergence into air allows an immediate determination of this incident chord; multiplying this measured value by n gives the size of the refracted chord; drawing in the refracted chord gives the direction of the light emerging into air.

(7) This same path is then merely traced on the back side of the second prism diagram, and the two sheets are joined together in this fashion:

(8) Vertical lines are drawn joining the prism bases and a horizontal line is drawn through the center of the prism system; beside identifying an axis of the prism system, this line could represent the undeviated path of a ray incident along the normal.



(9) The effect of A for prisms of the same n is shown graphically by aligning the centers of the three prism systems along a vertical line (a diagram appears on a separate page).

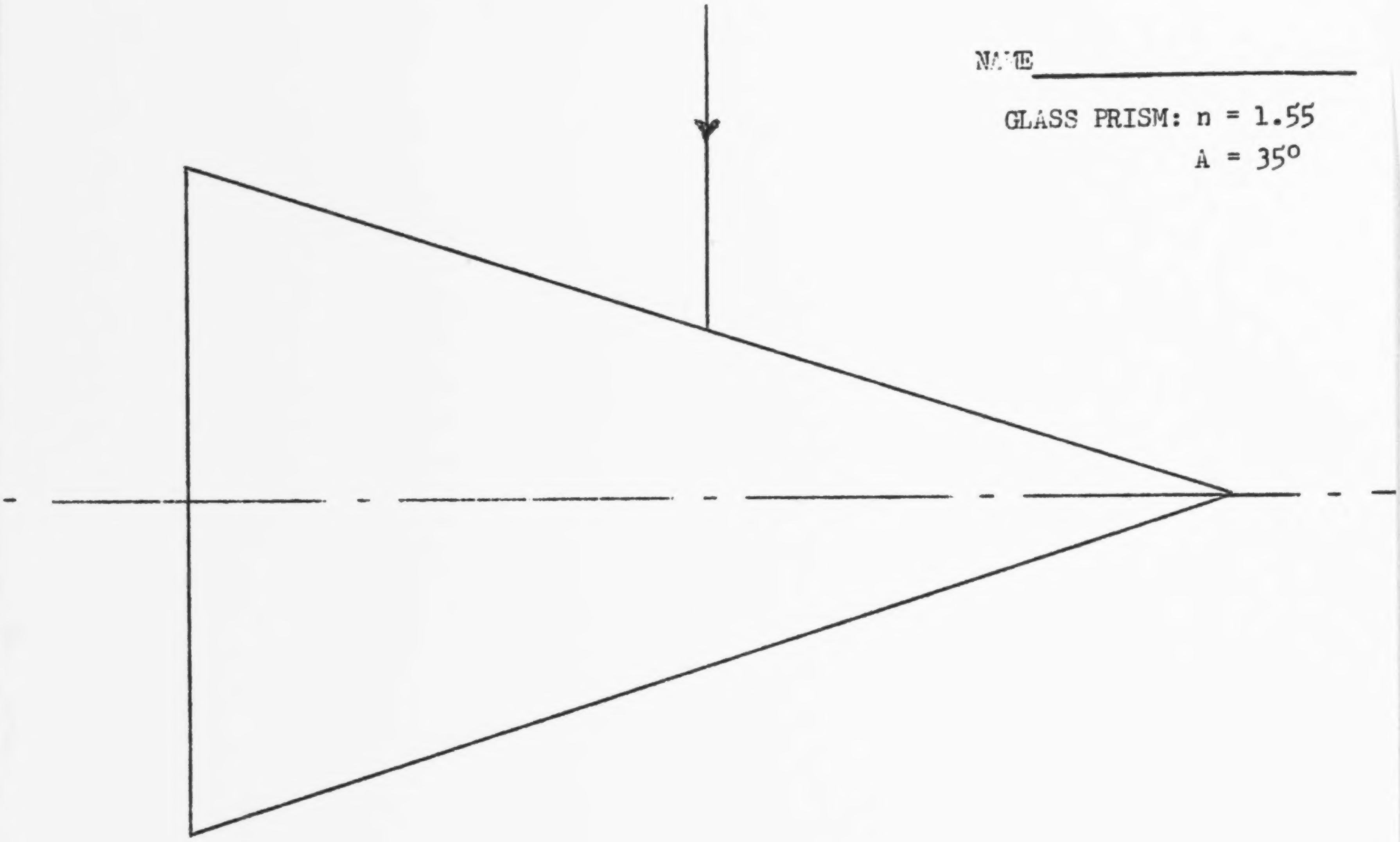
(10) Students are next instructed to introduce additional, symmetrical, parallel rays incident on the prism faces, one pair closer to the apex and the second pair closer to the base; their paths through the prism parallel the ones already determined.

(11) By this time, it is usually evident that the converging lens can be evolved from such a system.

(12) For an experimental check: prism system of ½" clear Lucite are easily made with the appropriate apex angles; these can be aligned vertically and mounted on a board; a light with adjustable slits provides the incident parallel rays; a rather striking demonstration of the effect of A results.

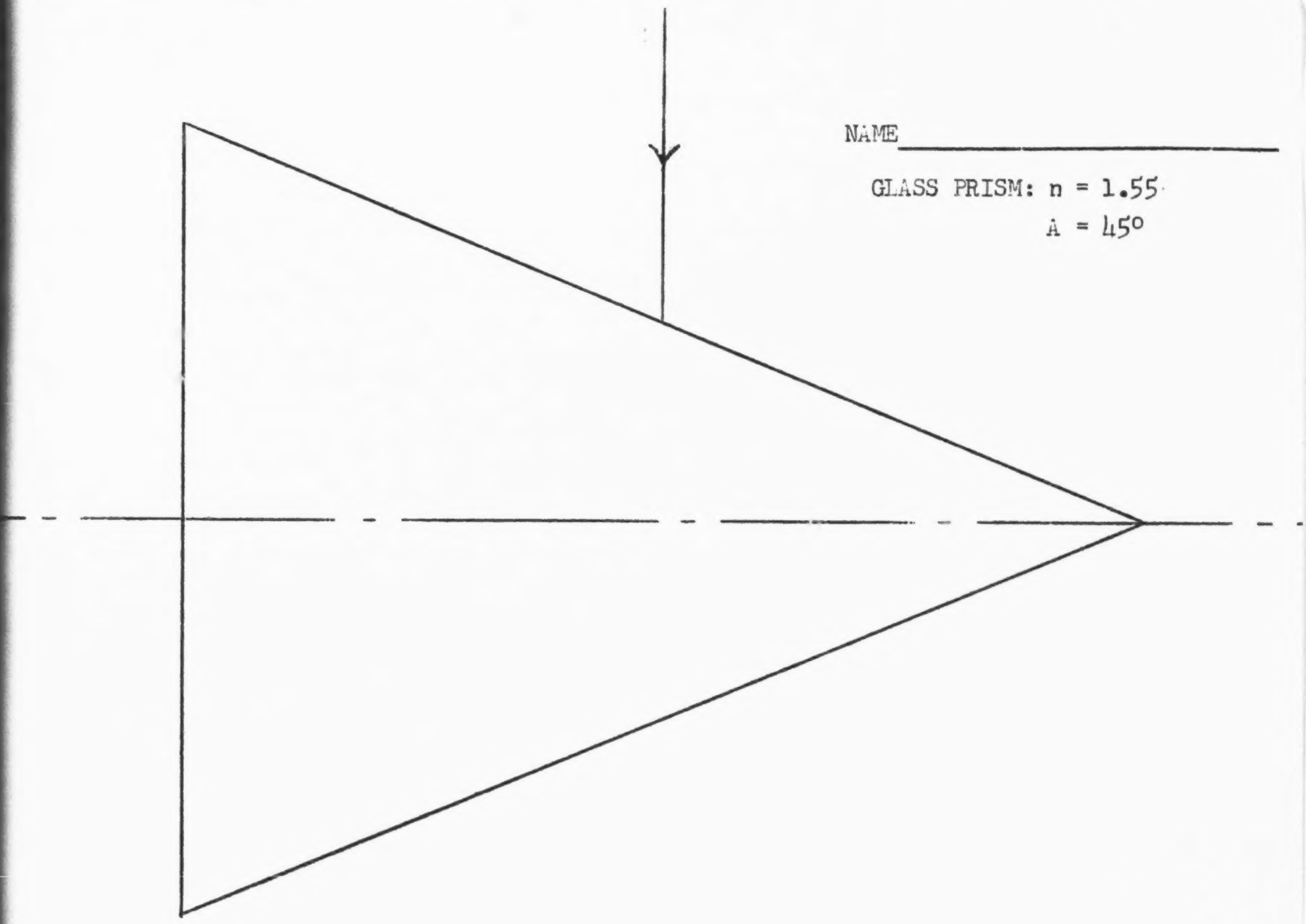
NAME ______________________

GLASS PRISM: n = 1.55
A = 35°



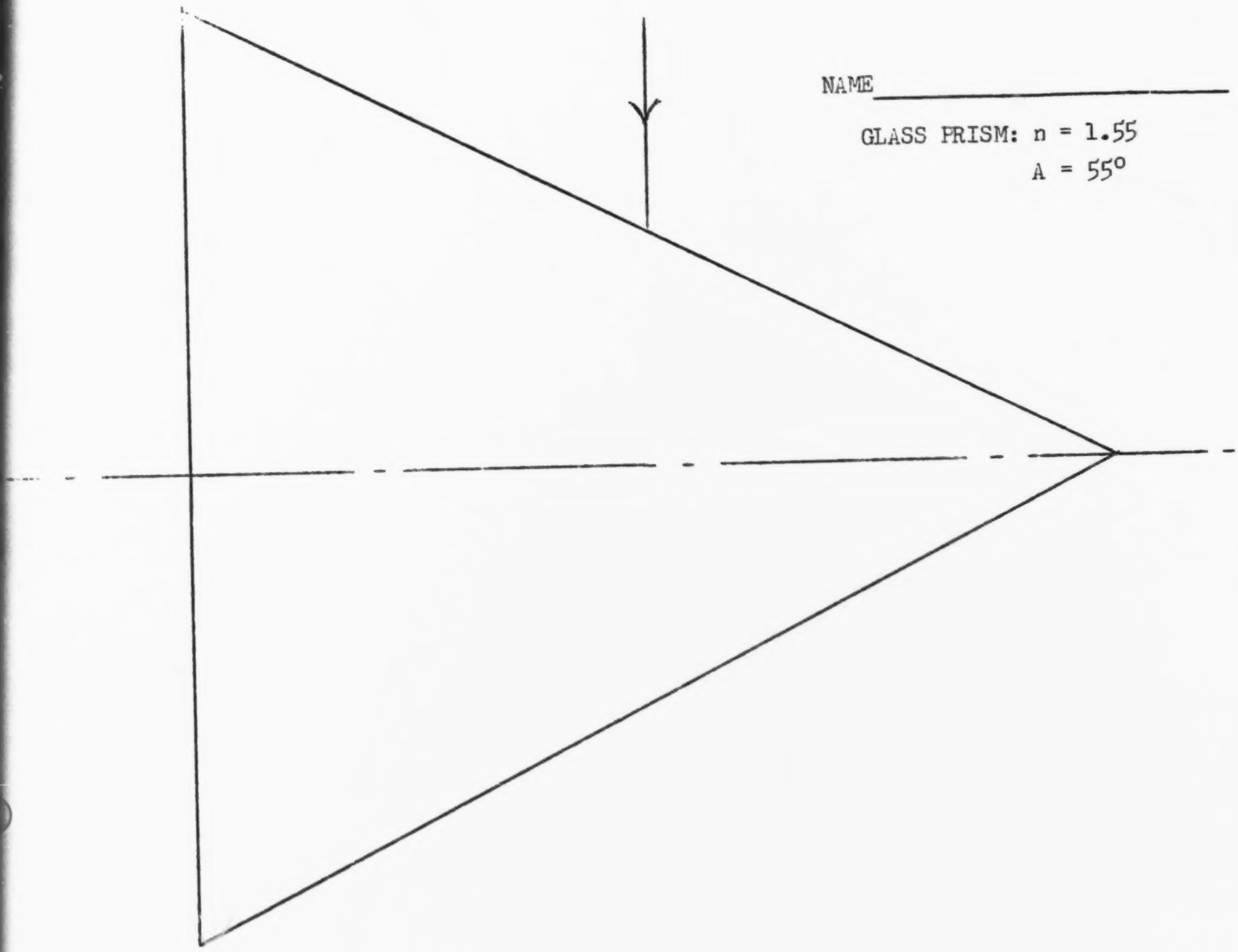
NAME______________________________

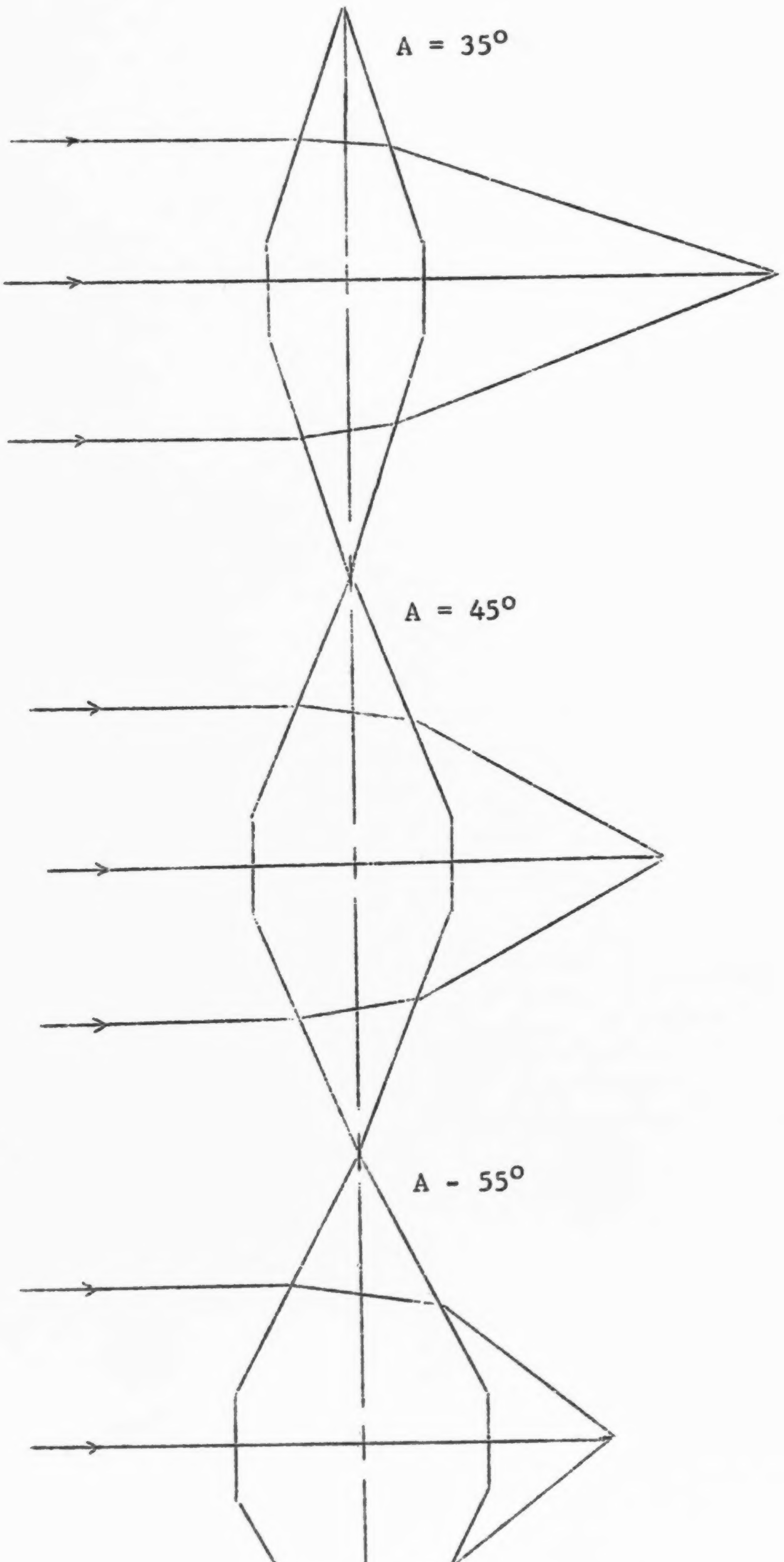
GLASS PRISM: n = 1.55
A = 45°



NAME________________________

GLASS PRISM: $n = 1.55$
$A = 55^\circ$



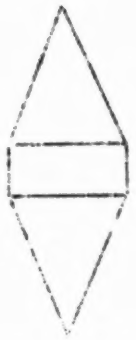

A = 35°
A = 45°
A - 55°

UNIT V. REFRACTION OF LIGHT

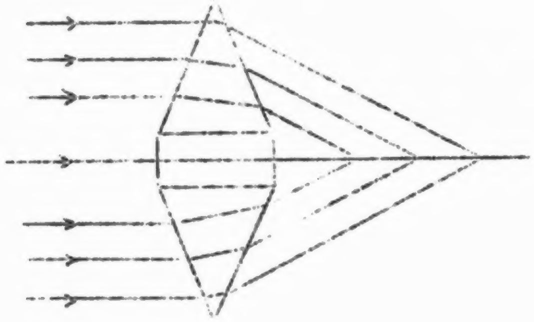
E. Convergence of a parallel beam of light by refraction (continued)

4. Evolution of the CONVERGING LENS

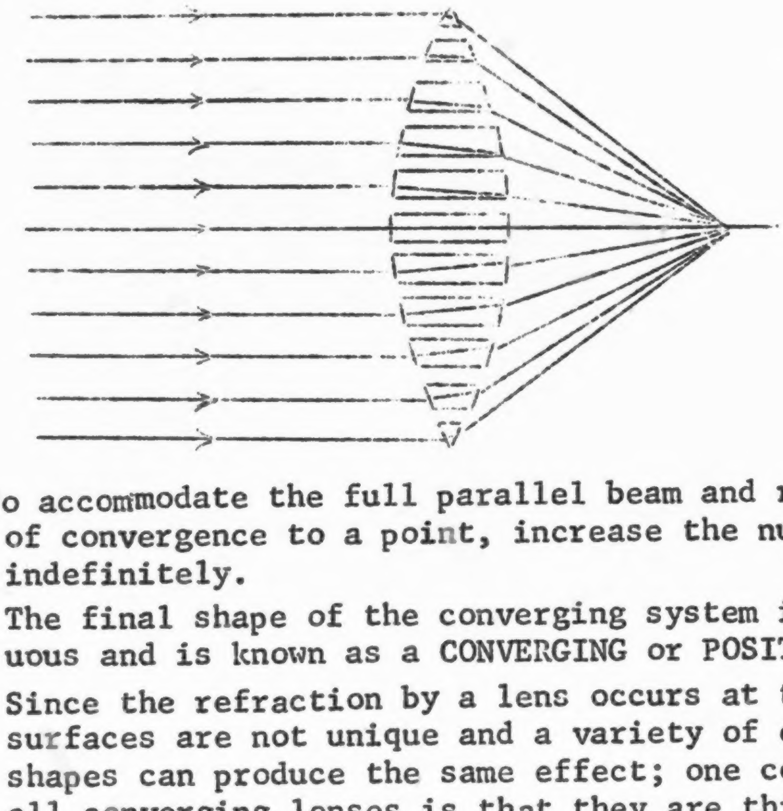
   a. To converge a beam of parallel light to a chosen point, start with a prism system (two like prisms, base to base) of a given n and A.



   b. Each pair of symmetrically spaced rays of a parallel beam incident on the prism system will be deflected to points a different distance from the prism.



   c. Cut the prism system into segments having the proper projected apex angle to direct the refracted light to the chosen place of convergence.



   d. To accommodate the full parallel beam and reduce the region of convergence to a point, increase the number of segments indefinitely.

   e. The final shape of the converging system is smooth and continuous and is known as a CONVERGING or POSITIVE LENS.

   f. Since the refraction by a lens occurs at two interfaces, the surfaces are not unique and a variety of combinations of surface shapes can produce the same effect; one common characteristic of all converging lenses is that they are thicker in the middle than at the ends.

   g. The point to which the parallel beam is converged is called the PRINCIPAL FOCUS F and the distance from the center of the lens to F is called the FOCAL LENGTH f.

UNIT V. REFRACTION OF LIGHT (continued)

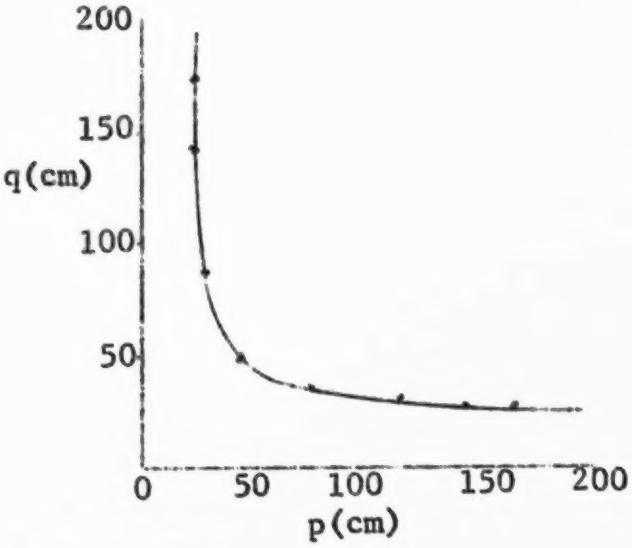
F. Formation of a real image by a converging lens

1. Laboratory: use the meter-stick optical bench to investigate the real image. LABORATORY

   a. Measure and record the focal length f of the lens.

   b. Vary the object distance p (measured from the center of the lens) and note the corresponding image distance q (also measured from the center of the lens).

2. Plot q vs p.



   a. The resulting q vs p graph resembles a hyperbola of the form y = k (1/x).

   b. A quick check of products (or a plot of q vs z, where z = 1/p) reveals it is not of the form y = k (1/x), nor is it y = k $(1/x^2)$.

   c. The features of the curve are similar to those of the curve obtained for the converging mirror, which through x_o and x_i eventually led to the equation, 1/p +1/q = 1/f.

QUANTITATIVE SCIENCE DATE______________ NAME____________________

1. In the experiment on the formation of a real image by a converging lens,
   a. Take FIVE sample products $x_i x_o$; compare each with the square of the measured focal length; calculate the percent difference.
   b. For five sample data points, find the linear magnification M; in one column insert values of M by computing h_i/h_o, and in another, q/p. Compute the percent difference.
   c. Plot 1/q vs 1/p. Compare to the converging mirror case. Conclude.

2. By using principal rays 1 and 2, show how a converging lens forms a virtual image.

3. The properties of a converging or positive lens have led to a host of applications, ranging from simple magnifiers to highly complex scanning computers. Using appropriate source materials, report on the design and operation of the following optical refracting instruments:
   a. Eye.
   b. Camera and enlarger.
   c. Simple microscope.
   d. Compound microscope.
   e. Telescope.
   f. Slide projector or Projection Lantern.

4. Show that the linear magnification M of a simple magnifier is given by the expression

   $$M = \frac{25(\text{cm})}{f} + 1,$$ where 25 (cm) is the distance of most distinct vision.

5. A full-length picture of a 5'4" model is to be taken with a camera whose film size is 4" X 5". If the distance from the lens to the film is 8", how far from the lens must the model pose?

*6. The objective lens of a telescope has a focal length of 1.0 meter. If the eyepiece focal length is 2.0 cm, what is the magnification?

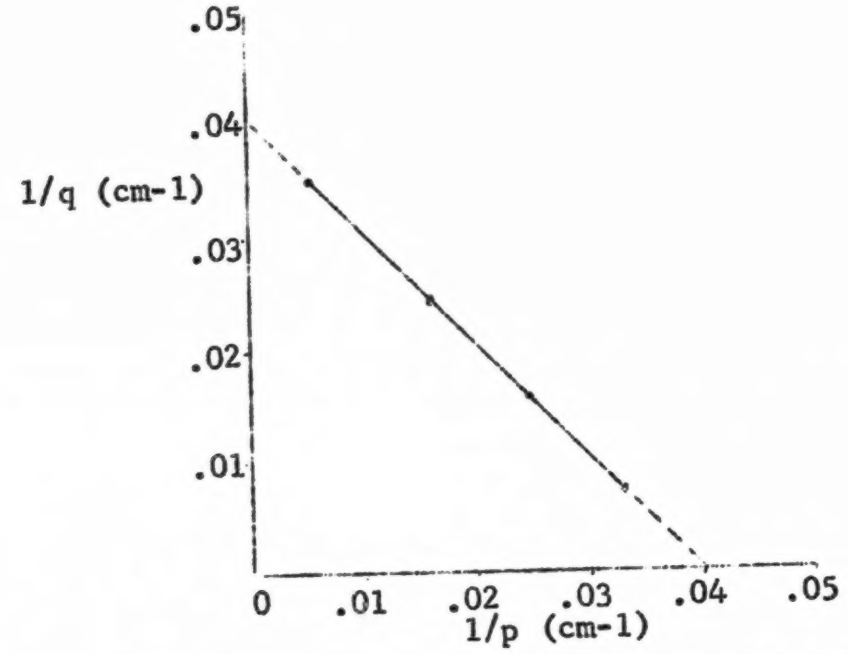
*7. The tube length of a compound microscope is 15 cm. If the focal lengths of the objective and eyepiece is 3.0 mm and 20 mm, respectively, what is the magnification?

*8. If the focal length of a camera is 80 mm, what is the diameter of the aperture at a setting of f/22?

* Use class and library reference materials.

F. Formation of a real image by a converging lens (continued)

3. Acting boldly, instead of introducing the x_o and x_i formulation here, plot a graph of the reciprocals, that is, $1/q$ vs $1/p$.



   a. The $1/q$ vs $1/p$ graph is a STRAIGHT LINE OF NEGATIVE SLOPE NOT THROUGH THE ORIGIN.

   b. Since it is a straight line, it must be of the form $y = mx + b$.

   c. y is $1/q$, x is $1/p$, so $1/q = -m\,(1/p) + b$.

   d. Compute the slope: note that it is dimensionless and has a value very close to -1.

   e. Read the value of b from the graph: note that it has units of reciprocal length; compare the value of b to the reciprocal of the focal length.

   f. Gather all this together: $y = mx + b$ has become $1/q = -1(1/p) + 1/f$, and rearranging, $1/p + 1/q = 1/f$, an amazing result!

4. This expression obtained for a converging lens, $1/p + 1/q = 1/f$, is referred to as the THIN LENS EQUATION.

   a. Of course it is identical to the MIRROR EQUATION.

   b. What a marvel of nature that two entirely different physical phenomena (reflection and refraction) are described by an identical mathematical formulation!

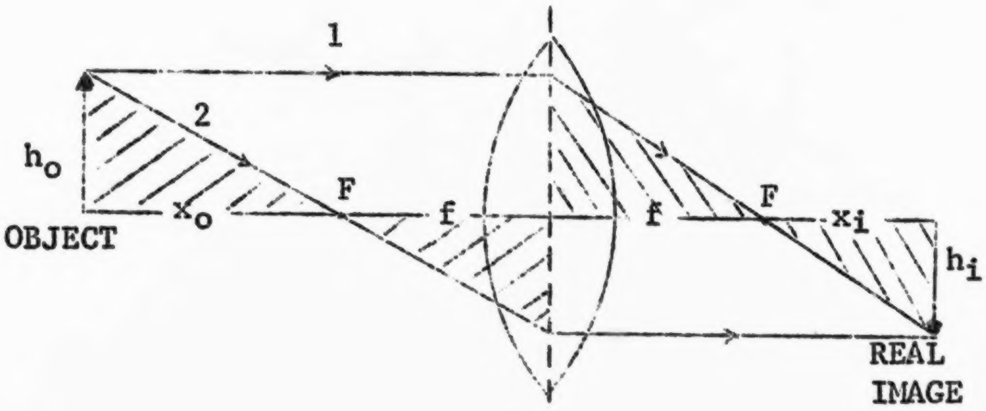
F. Formation of a real image by a converging lens (continued)

5. Geometry of real image formation

   a. Place an object h_o in height any distance $p > f$ from the center of the lens; it is then a distance x_o from the principal focus F.

   b. Even though light suffers two direction changes in passing through a lens, it is simpler to regard the net change as due to a single refraction, taking place along a line drawn through the lens center, perpendicular to the principal axis.

   c. This introduces what is known as the THIN LENS APPROXIMATION and will be used in the analysis to follow.



   d. PRINCIPAL RAY 1 is incident in a direction parallel to the principal axis and hence is refracted through F on the other side of the lens.

   e. PRINCIPAL RAY 2 is incident in a direction through F and hence is refracted parallel to the principal axis on the other side of the lens.

   f. The intersection of these rays originating from the same object point forms the corresponding point on the image.

   g. Note that P - Type Similar Triangles are formed, so $h_i/h_o = f/x_o$, and $h_i/h_o = x_i/f$.

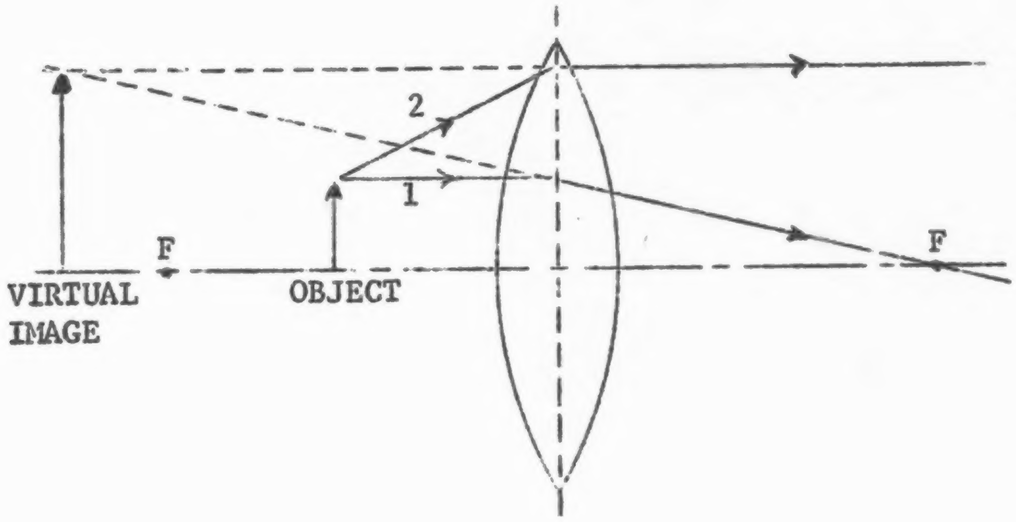
   h. Again giving $x_i x_o = f^2$, which has already been shown equal to $1/p + 1/q = 1/f$.

UNIT V. REFRACTION OF LIGHT

G. Formation of a virtual image by a converging mirror

1. Observe that no real image is formed by a converging lens if the object is placed inside the principal focus; for $p < f$, only an erect, enlarged, virtual image is formed ON THE SAME SIDE OF THE LENS AS THE OBJECT.

2. Apply geometry similar to that used in the case of the virtual image formed by the converging mirror to predict the virtual image formed by the converging lens.



H. Problems, discussion, and special topics in refraction

1. Magnification

2. Refracting instruments

EXPERIMENTS

   a. Eye

   b. Camera

   c. Simple microscope

   d. Compound microscope

   e. Telescope

QUANTITATIVE SCIENCE DATE________________ NAME________________________

Mark each statement TRUE or FALSE in the blank at the right of each:

1. A lens with two convex surfaces has the ability to converge parallel light. 1______
2. For light going from air to a substance, the higher the index of refraction, the greater the angle of refraction. 2______
3. The focal length of a lens is the distance from the lens surface to the principal focus. 3______
4. Glass refracts more than water. 4______
5. The index of refraction of a substance is given by the slope of the line obtained in plotting the chord in the substance vs the chord in air. 5______
6. A spherical drop of water will refract light. 6______
7. The index of refraction of a particular prism depends in part on the apex angle. 7______
8. The index of refraction of a given substance and its reciprocal when added together gives one. 8______
9. Light striking the surface of a 45° prism along the normal will not be refracted by the prism at all. 9______
10. Light from air through a rectangular glass slab back to air remains unchanged in path. 10______

Select from the LIST below the letter of the word or phrase which best matches or describes the numbered STATEMENT:

LIST

A. interface
B. angle of refraction
C. H. Snellen
D. focal point
E. indigo
F. optical prism
G. angle of incidence
H. index of refraction
I. Tintex
J. n and A
K. Descartes
L. fluorescein
M. W. Snell
N. angle of deviation
O. optical disc
P. principal focus
Q. converging lens
R. Rit

STATEMENT

11. Transparent object whose sides are not parallel. 11______
12. Two quantities which in combination determine the angle of deviation of a prism. 12______
13. Boundary between two dissimilar media. 13______
14. Angle which measures the departure of a light path from its original incident direction. 14______
15. Man who quantitatively first described refraction15______
16. Dye used in class to render the path of light in water visible. 16______
17. Instrument used in class to study the refracting properties of glass, or plastic. 17______
18. Point at which the light from a parallel beam is concentrated by a converging lens. 18______
19. Num ber which describes the ability of a transparent substance to change the direction of the light entering or leaving it. 19______
20. Refracting object which has the ability to bring all the light from a parallel beam to a single focus. 20______

QUANTITATIVE SCIENCE DATE________________ NAME________________________

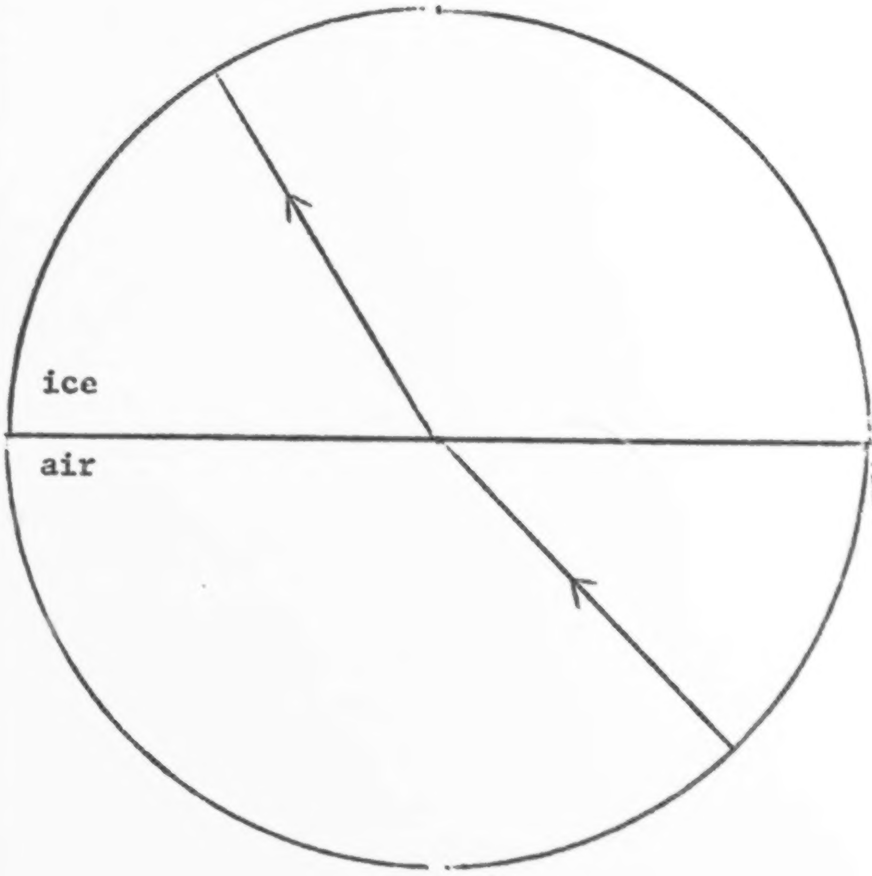
21. The following diagrams represent the paths of light in various substances. From the diagrams, determine the index of refraction of:

a. Ice

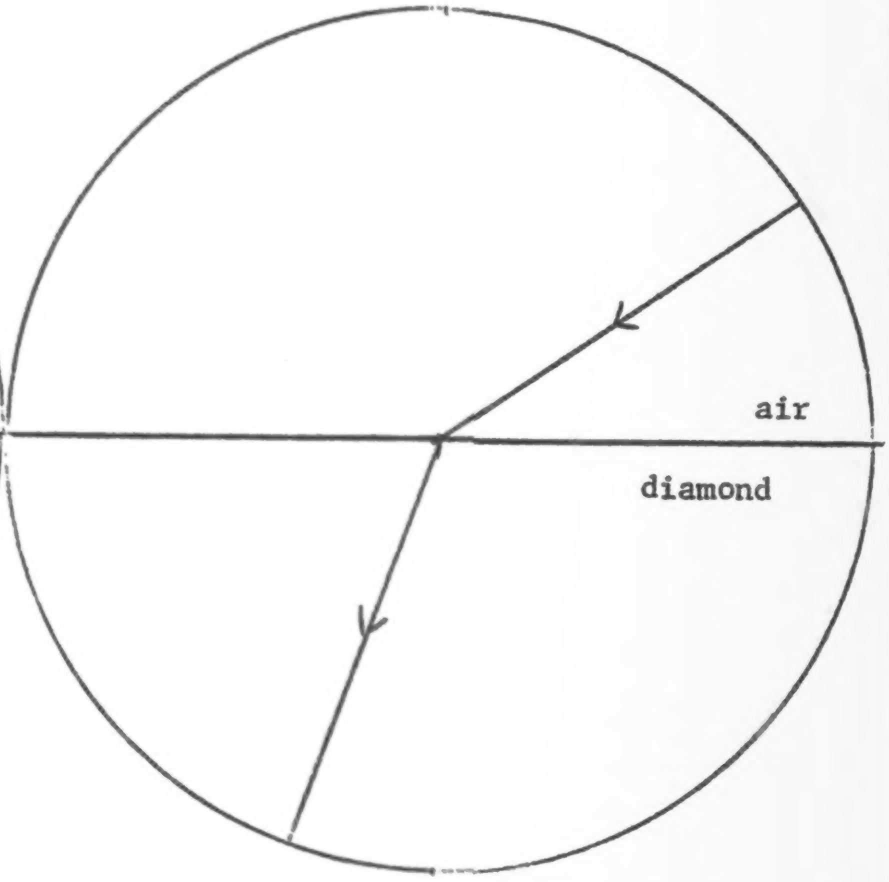
b. Diamond

c. Lucite

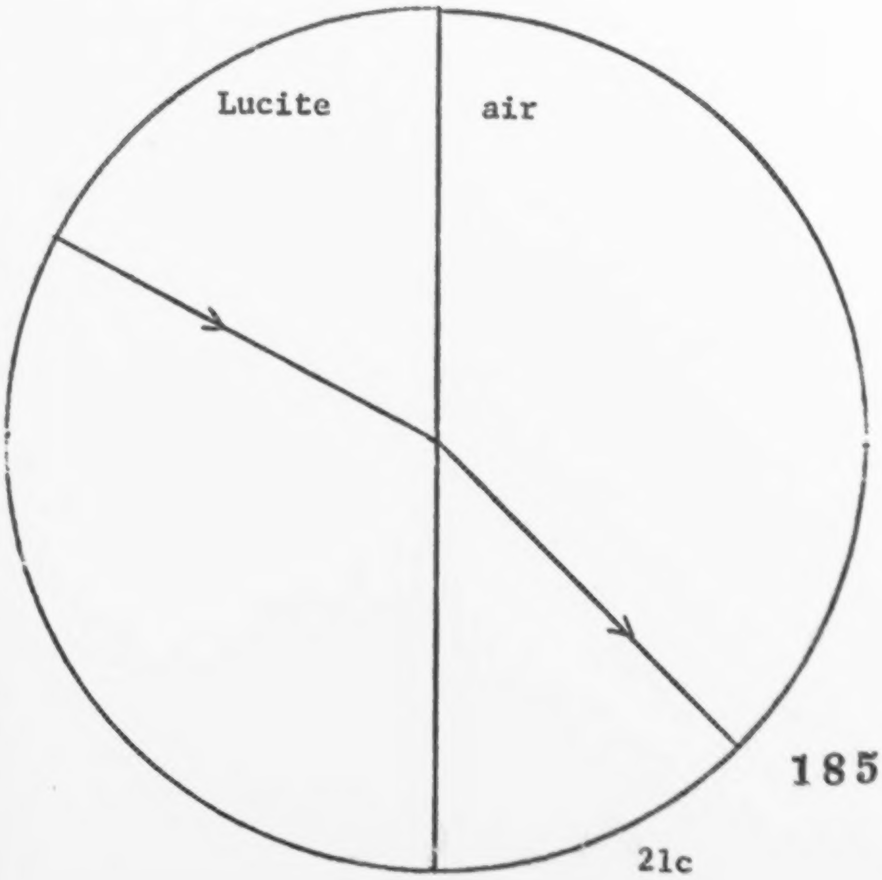
SHOW ALL WORK



21a



21b



21c

21 a ____________

21 b ____________

21 c_____________

QUANTITATIVE SCIENCE DATE________ NAME______________

Select from the LIST the item which is best described by the STATEMENT. Use no item more than once and use no more than one item per statement.

LIST

A. $p < f$

B. $p > f$

C. eyepiece

D. pupil

E. optic nerve

F. telescope

G. positive lens

H. microscope

I. crystalline

J. presbyopia

K. retina

L. myopia

M. virtual

N. real

O. astigmatism

P. ciliary

Q. lenspower

R. cornea

S. hyperopia

T. focal length

U. iris

V. prism

W. f-stop

X. 10 inches

Y. diopter

Z. $p = f$

STATEMENT

1. Optical instrument in which the objective lens is one of short focal length. 1________

2. Muscle which controls the shape of the eye's lens in forming an image. 2________

3.Lens which uses the real image formed by another lens as its object in producing a virtual image. 3________

4. "Screen" of the eye on which the lens forms an image. 4________

5. Distance of the object from the objective lens of a compound microscope. 5________

6. Quantity (in thin lens equation) which changes when eye adjusts to focus on a distant object. 6________

7. Optical instrument in which the objective lens is one of long focal length. 7________

8. Kind of image formed by objective lens of a compound microscope. 8________

9. Specific name of the image-forming lens of the eye. 9________

10. Kind of image formed by the eyepiece lens of an astronomical telescope. 10________

11. Transparent object whose sides are not parallel. 11________

12. Refracting object which converges a beam of parallel light. 12________

13. Ratio of the focal length of a lens to the diameter of the aperture. 13________

14. Average value for the distance of distinct vision. 14________

15. Defect of the eye in which an image is focused behind the retina. 15________

QUANTITATIVE SCIENCE DATE_______________ NAME___________________________

16. DESCRIBE, with some quantitative detail and diagrams if necessary or helpful, the EVOLUTION OF A CONVERGING LENS, beginning with light incident on a prism.

17. a. Show by the geometry of image formation by a positive lens (using the similar triangles formed by principal rays 1 and 2) that $x_i x_o = f^2$.

b. Relate this expression to the results of the experiment with lenses and the thin lens equation.

18. The lens of a popular camera has a focal length of 50 mm. A picture is to be taken of an object 1.5 meters in height.

a. How far from the lens is the film if the object is placed at a distance of 2.0 meters in front of the camera?

18a____________________

b. What is the height of the image?

18b____________________

c. If the camera is set at f/11, what is the effective diameter of the aperture?

18c____________________

Instructor Information Sheet

NOTE ON EXPERIMENTS IN UNIT VI.

Many of the laboratory activities suggested are such that they can be used to demonstrate several principles advanced in this unit. It is consequently difficult to assign a given experiment to a particular section of subject matter. As an example: treated quantitatively, the results of the water-to-ice phase change gives evidence for the following:

(1) VOLUME as a (poor) measure of matter.

(2) CHANGE OF STATE as a physical change.

(3) Volume EXPANSION as a physical change.

(4) CONSERVATION OF MASS in physical change.

(5) CONSERVATION OF MASS in change of state.

(6) DEPENDENCE OF STATE on temperature.

(7) Change in physical properties: hardness, refractive index, etc.

(8) Change in density.

(9) Melting of ice as reference point for temperature measurement.

(10) CHANGE OF STATE and the Particle Model.

Since the total subject matter in this unit is not very extensive, one way to maximize this advantage of overlapping significance of the experiments is to spend considerable time in the laboratory initially, with the students carefully and completely recording observations in a variety of experiments for later recall and reference in discussion.

Student interest in these chemical investigations generally runs rather high ---capitalize on it!

UNIT VI. Matter: INTRODUCTION TO CHEMISTRY

A. Matter

1. Definitions
   - a. Anything that occupies space and has mass.
   - b. Substance of which a physical object is composed; "stuff" of the universe.
   - c. "Substance that constitutes the observable universe and together with energy forms the basis of objective phenomena" --- from a college chemistry textbook.
   - d. Philosophical: matter and form.

     EXAMPLE: man --- flesh is matter, spirit is form.

     EXAMPLE: apple --- pulp, water, seeds, skin, stem, etc., constitute the matter; "appleness", the form.

2. Substance
   - a. Specified matter.
   - b. Any one particular kind of matter; mark of homogeneity.
   - c. Matter of a particular or definite chemical composition.
   - d. EXAMPLE: chunk of granite; close inspection reveals dark specks, beady fragments, and a grayish, translucent material; the three different constituents can be separated by mechanical means; granite is a MIXTURE of three SUBSTANCES, which are identified as mica, quartz, and feldspar, respectively. EXPERIMENT

3. Measure of matter
   - a. Volume as a measure of matter.

     EXAMPLE: Equal volumes of gases have different masses. DEMONSTRATION*

     EXAMPLE: Water changing to ice changes volume while the mass remains constant. EXPERIMENT*

     EXAMPLE: Liquids expand with an increase in temperature but the mass remains constant. EXPERIMENT*
   - b. Mass as a measure of matter: the MILLIBALANCE EXPERIMENTS**
   - c. Principle of Conservation of Mass: MATTER CAN NEITHER BE CREATED NOR DESTROYED. EXPERIMENTS*

4. Matter occurs, that is, substances are observed as ELEMENTS, as COMPOUNDS, or in MIXTURES.

5. Elements
   - a. Substances that cannot be decomposed into simpler substances, nor can be produced by any combination of other substances.
   - b. Substances which go through chemical manipulation without being resolved into simpler substances.
   - c. Substances which cannot be decomposed in the ordinary types of chemical change, or made by chemical union --- from HANDBOOK OF CHEMISTRY AND PHYSICS.

Experiments: VOLUME AS A MEASURE OF MATTER

In addition to the observations on volume expansion in a change of state, a study of volume changes within each of the three states of matter is thought to be of some merit.

LIQUID CONTRACTION:

(1) 5- and 10- ml graduate cylinders are usually small enough that they can be placed on a balance; determine the individual mass of two graduates, at least one being the 10-ml size.

(2) Fill each graduate carefully to the 5.0 ml mark with water; measure the mass of water in each.

(3) Pour the water from one (label this one transfer) graduate to the other (standard) and record accurately the combined volume; the purpose of the procedure to this point is to "calibrate" the graduate cylinders (i.e., a reading of 5.0 ml on the transfer graduate adds x ml to the reading on the standard graduate).

(4) For the actual measurements in the experiment, determine the mass of exactly 5.0 ml of water in the standard graduate, and 5.0 ml of alcohol (isopropyl and ethyl work equally well) in the transfer graduate.

(5) Carefully combine the alcohol with the water; record the mass and volume of the combined liquids in the standard graduate.

(6) Compare the mass before and after combining the liquids; compare the volume to the calibrated 10.0 ml of the standard graduate and compute the percent difference, which gives the volume change.

(7) The procedure can be refined somewhat by the use of a pipette for the liquid transfer, if it is desired to introduce this technique.

LIQUID EXPANSION

(1) Discarded milk-test bottles (with calibrated narror neck) have provided an excellent quantitative study of volume expansion in various liquids.

(2) The bottles are filled to the marked zero on the neck and placed in a large, water-filled beaker which provides a controlled temperature bath.

(3) As the bath is heated, the liquid rise in the narrow neck is dramatic and yields excellent data for the VOLUME (initial volume + increase) vs TEMPERATURE graph and the consequent discussion of the coefficient of expansion.

(4) Pointing out that the mercury in the thermometer is doing the same thing as the liquids in the bottle necks opens the way for the construction and calibration of a thermometer, either with a commercially available ungraduated model (WELCH SCIENTIFIC) or student-made models.

Experiments: VOLUME AS A MEASURE OF MATTER (continued)

EXPANSION OF SOLIDS:

(1) The traditional linear expansion apparatus serves effectively as either a demonstration or student experiment.

(2) The LENGTH vs TEMPERATURE graph here again can be used to initiate discussion on the coefficient of linear expansion.

(3) Quantitative comparison to table values of the expansion coefficient can also be made.

(4) Later, after the Particle Model is introduced, the expansion of solids can be interpreted on the basis of this Model.

VOLUMES OF GASES:

(1) CHEMS Experiment 6 is suggested as a demonstration.

(2) Boyle's and Charles' Laws (although not necessarily identified by those names) further illustrate volume measure in gases; the "Moe Tube", AMERICAN JOURNAL OF PHYSICS 26, page 35, January 1958, is worth pursuing for this purpose since both the Boyle's and Charles' Law data can be obtained with relative ease on the same piece of apparatus.

Experiment: WATER-TO-ICE CHANGE

Quantitative observation of the volume expansion in the change of state is of primary concern here, but as noted elsewhere, this experiment can serve to illustrate other phenomena as well.

Graduated plastic hypodermic syringes are safe and quite satisfactory for this activity. Used, disposable, 2-ml capacity syringes can easily be obtained free of charge. While the syringe offers several operational advantages, a graduated plastic pipette or even ordinary plastic tubing properly marked can probably be used as well.

(1) Seal the syringe needle or stem: after measuring the mass of the empty syringe, carefully add distilled water to exactly the 1.0 ml mark (or some other convenient level)

(2) Lower the syringe piston until it barely touches the water surface; take care to keep the syringe in a vertical position from now on since the piston fit is not always reliable.

(3) Determine the mass of the water; this can be done by suspending the syringe from the balance pan hanger.

(4) Place the syringe in a freezer; reading the initial temperature of the water (usually near room temperature) and the freezer temperature may be suggested.

(5) After the water has frozen (the next class meeting is a good time) read the ice volume on the syringe barrel and quickly check the mass for any change.

(6) Calculate the per cent change in volume as water changes to ice; comment on the mass of water during the expansion; relate this to the density (later?)

(7) Be prepared for questions on possible expansion or contraction of the plastic syringe, the effect of water condensation on the cold syringe surface, etc.

(8) Initiate discussion on volume as a measure of mass.

QUANTITATIVE SCIENCE NAME________________________

A glass tube whose inside diameter is 2.0 mm is sealed at one end and then filled to a depth of 10.00 mm with a liquid carbon disulfide (CS_2) at a temperature of 0°C. It is then gently heated and the liquid depth (or length of liquid column L) is measured as a function of temperature T.

T (°C)	L (mm)
0	10.00
10.0	10.11
20.0	10.22
30.0	10.34
40.0	10.46
50.0	10.57

a. Plot the VOLUME vs TEMPERATURE.

b. From the graph: what is the volume at 15°C?

c. What does the graph predict for the volume at 60°C?

d. Analyze the graph to write the expression relating the VOLUME and TEMPERATURE.

QUANTITATIVE SCIENCE DATE________________ NAME________________________

SENSITIVE MILLIBALANCE

In essence, this sensitive balance is a device having a single beam (soda straw) pivoted (straight pin on a glass "knife edge") off center (adjustable counterweight at short end of the beam) accepting loads on a pan (aluminum foil) suspended by silk thread, and having an arbitrary marked scale calibrated against mass units called "margs".

CONSTRUCTION

Base: wood (metal or other material if desired), about 10 inches long, 5/8" to 1" in height and width.

Knife edge: (beam support) glass microscope slides, metal, marked or notched support position; may be taped on, firmly.

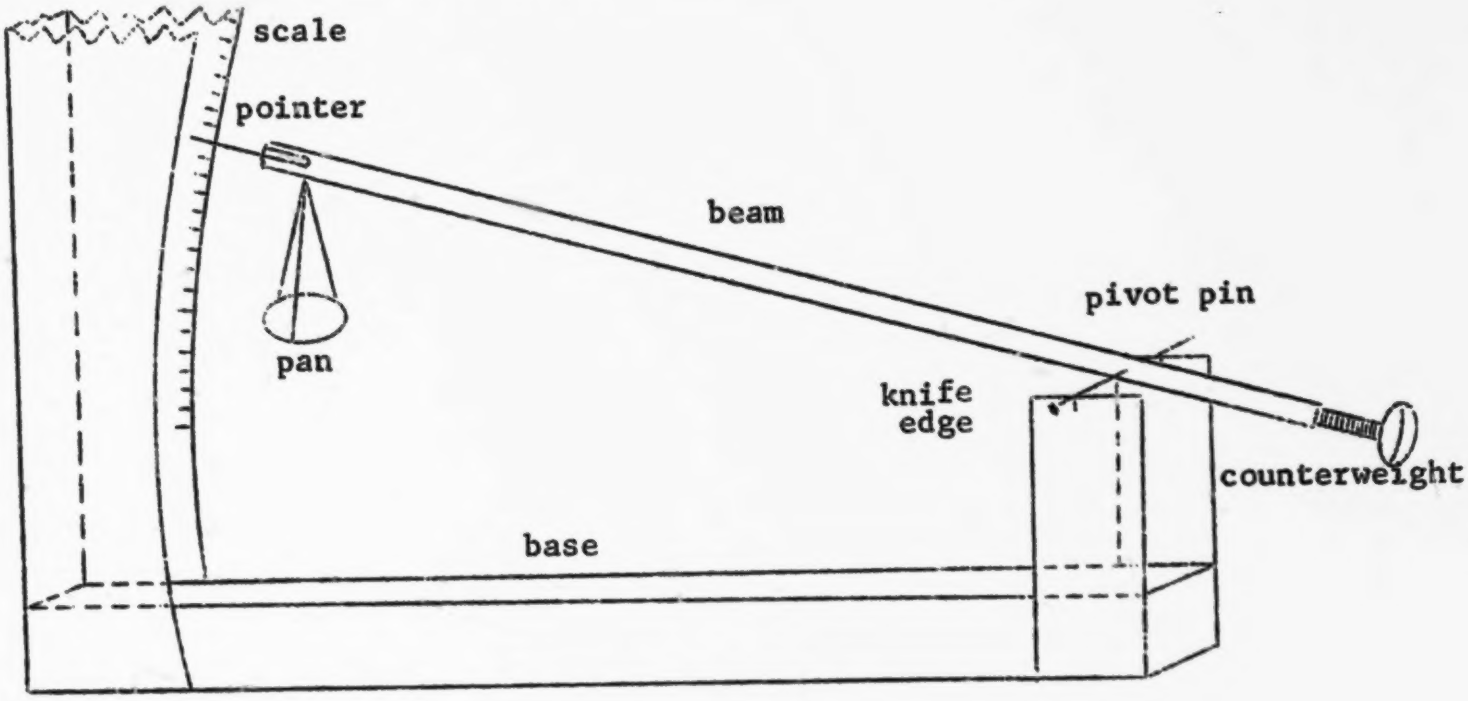
Counterweight: machine screw of proper size to fit the straw snugly, permitting advance of screw into straw length by turning.

Balance pan: shaped (round or square flat bottom) out of aluminum foil with bottom slightly greater than 1" along a single dimension; suspended by 3 - 4 thin silk threads attached at the load end of the soda straw.

Pivot pin: straight pin inserted slightly above center line down length of the straw; pin position determined by locating the center of gravity of the straw with the empty pan mounted on one end and the counterweight on the other.

Scale: cardboard or stiff paper cut into an arc of a circle whose radius is determined by the length from the pivot pin to the load end of the straw.

Pointer: paper clip used as an indicator; attached at load end of the straw in a position to ride along scale.



CALIBRATION

Adjust the counterweight so that the beam makes an angle of about 45° above the horizontal. Mark this pointer position "0" on the scale. A protractor may be used to lay off the scale from 0 down to 45 or 50 divisions.

Starting with the balance "zeroed", add one marg at a time and obtain data for a CALIBRATION CURVE of scale divisions vs margs.

Experiment: USE OF THE MILLIBALANCE

A few simple, direct mass measurements are in order after the millibalance is contructed and a calibration curve (scale divisions vs margs) is obtained. The student is merely directed to use his millibalance to determine the mass (in margs) of the following:

(1) A postage stamp (or a trading stamp).

(2) A conduct slip or corridor pass, or other item of local interest (the idea is to choose an object that gives something more than full-scale deflection so that the mass of two or more pieces will have to be determined and added for the total; the thing to watch for is that margs be added, and not scale divisions); it would probably be interesting, but it is illegal to cut a dollar bill...

(3) A piece of aluminum foil (about 2 cm on a side); properly shaped, this can be used as a container for the water in the next part.

(4) A drop of water --- and additional drops to the limit of the foil container or full deflection of the balance, whichever occurs first.

QUANTITATIVE SCIENCE DATE_______________ NAME___________________________

THESE PROBLEMS ARE TO BE WORKED IN YOUR LABORATORY NOTEBOOK AND EACH SHOULD BE CLEARLY IDENTIFIED.

1. 136 margs used in the calibration of the millibalances were placed on an ANALYTICAL BALANCE (Mettler) and found to have a mass of 0.6256 grams.
    a. What is the mass of one marg?
    b. What is the mass of 10^6 margs?
    c. How many margs are contained in one kilogram?
    d. What mass (in grams) gives full-scale deflection on your millibalance?
    e. Express the sensitivity of your millibalance in milligrams per scale division
    f. How many margs would weight one pound?

2. Refer to the first experiment performed with the millibalance.
    a. What is the mass (in grams) of a postage stamp? Be sure to identify the stamp.
    b. What is the mass (in grams) of a conduct slip?
    c. What average mass value (in grams) does the millibalance give for one drop of water?

3. Use a laboratory balance to determine the mass of 20, 40, and 60 drops of water, respectively; measure the volume (cm^3 or ml) of the 20, 40, and 60 drops of water, respectively, using a small graduate cylinder (or a pipette).
    a. Plot the measured mass (grams) vs the number of drops.
    b. Use the slope of the line to find the average mass value for one drop of water.
    c. Compare this result to that obtained with the millibalance.
    d. Plot the measured mass (grams) vs the measured volume (cm^3 or ml) for the 20, 40, and 60 drops of water.
    e. What is the significance of the slope of the line? What are the units of the slope?
    f. State a "rule of thumb" for the number of water drops in one cm^3 (or ml) based on the results of this investigation.

QUANTITATIVE SCIENCE NAME ______________________

MOST ABUNDANT ELEMENTS IN EARTH'S CRUST

Aluminum	Al	Molybdenum	Mo
Antimony	Sb	Neon	Ne
Arsenic	As	Nickel	Ni
Bismuth	Bi	Nitrogen	N
Boron	B	Oxygen	O
Bromine	Br	Phosphorus	P
Calcium	Ca	Platinum	Pt
Carbon	C	Potassium	K
Chlorine	Cl	Radium	Ra
Chromium	Cr	Silicon	Si
Copper	Cu	Silver	Ag
Fluorine	F	Sodium	Na
Gold	Au	Sulfur	S
Hydrogen	H	Tin	Sn
Iodine	I	Tungsten	W
Iron	Fe	Zinc	Zn
Lead	Pb		
Magnesium	Mg		
Manganese	Mn		
Mercury	Hg		

RELATIVE ABUNDANCE OF MOST COMMON ELEMENTS FOUND IN EARTH'S CRUST

(from Brooks, Tracy, Tropp: Modern Physical Science)

Oxygen	49.8%
Silicon	26.0%
Aluminum	7.3%
Iron	4.1%
Calcium	3.2%
Sodium	2.3%
Potassium	2.3%
Magnesium	2.2%
Other	2.8%

UNIT VI. MATTER: INTRODUCTION TO CHEMISTRY

A. Matter

5. Elements (continued)

d. Elements make up the distinct varieties of matter of which the universe is composed.

e. 103 (104?) elements now known; listed on PERIODIC TABLE OF ELEMENTS according to chemical behavior.

f. Name of element (from which its symbol usually follows) frequently denotes its origin or description:

EXAMPLES:	Scandium (Sc)	Scandinavia.
	Uranium (U)	Planet Uranus discovered about same time.
	Bromine (Br)	Greek, "bromos", stench!
	Sodium (Na)	Latin, "sodanum", headache remedy.
	Polonium (Po)	Poland.
	Gallium (Ga)	France.
	Germanium (Ge)	Germany.
	Niobium (Nb) vs	Columbium (Cb)

g. Elements beyond the first 92 have been "produced" since 1940:

93 Neptunium (Np)
94 Plutonium (Pu)
95 Americium (Am)
96 Curium (Cm)
97 Berkelium (Bk)
98 Californium (Cf)
99 Einsteinium (Es)
100 Fermium (Fm)
101 Mendelevium (Md)
102 Nobelium (No)
103 Lawrencium (Lw)
104 ?

h. Chemical symbols of elements; shorthand of chemistry. (see separate sheet of most common elements)

6. Compounds

a. Substances formed by the chemical composition of two or more elements

DEMONSTRATION

b. Substance containing more than one constituent element and having properties different from those its constituents had as elements.

c. EXAMPLES: water is a chemical combination of hydrogen (H) and oxygen (O).

quartz is a combination of silicon (Si) and oxygen (O).

rust is a combination of iron (Fe) and oxygen (O).

nitric acid is a combination of hydrogen (H), nitrogen (N), and oxygen (O).

QUANTITATIVE SCIENCE DATE________________ NAME______________________

IN THE SPACES AT THE RIGHT GIVE THE PROPER SYMBOLS FOR EACH OF THE ELEMENTS LISTED:

1. Boron 1.______________

2. Fluorine 2.______________

3. Potassium 3.______________

4. Tin 4.______________

5. Arsenic 5.______________

6. Manganese 6.______________

7. Tungsten 7.______________

8. Chromium 8.______________

IN THE SPACES AT THE RIGHT WRITE THE NAME OF THE ELEMENT WHOSE SYMBOL IS GIVEN:

9. Sb 9.____________________

10. Pb 10.____________________

11. Nb 11.____________________

12. Np 12.____________________

13. Cm 13.____________________

14. Mo 14.____________________

15. Md 15.____________________

UNIT VI. MATTER: INTRODUCTION TO CHEMISTRY

A. Matter

6. Compounds (Continued)

   d. QUANTITATIVE, EXPERIMENTAL, OBSERVATON: the composition of a given compound is perfectly and always the same, no matter how the compound is formed; there is a definite, unique, proportion of each element in a given compound. EXPERIMENT*

7. Mixtures

   a. Two or more substances intermingled with no constant percentage composition. EXPERIMENTS*
   b. Each substance (component) of mixture retains its original properties.
   c. Mixture may consist of elements, of compounds, or of elements and compounds.
   d. Mark of heterogeneity (except solutions).
   e. Components of mixture can be separated by physical or mechanical means.

8. Identification of substances: PROPERTIES of matter.

   a. Physical properties.
   b. Chemical properties.

B. Physical properties

1. Characteristics inherent to a substance or substances; properties that relate to the substance itself and are physically observable.

2. EXAMPLES: EXPERIMENTS*
   a. color, odor, taste.
   b. density, hardness, ductility, malleability.
   c. melting point, boiling point.
   d. solubility.
   e. heat expansion, heat conductivity.
   f. electrical conductivity.
   g. crystalline form.
   h. index of refraction.
   i. viscostiy.
   j. others

3. Physical change
   a. Any change which does not destroy the identity of a substance or substances.
   b. Any change which does not alter the chemical, composition of a substance or substances. EXPERIMENTS*
   c. A change which affects only the physical properties of a substance.
   d. EXAMPLES: copper rod expands when heated.
      steam condenses on a cold surface.
      ice melts when heated.
      viscosity of molasses changes with temperature.
   e. Mass remains constant during a physical change.

B. Physical properties (continued)

4. States of matter

   a. Matter exists in the form of SOLIDS, LIQUIDS, or GASES. (a fourth form the PLASMA, should also be listed even though it is not commonly encountered except in laboratory situations on earth).

   b. Solids: definite mass, volume, shape.

   c. Liquids: definite mass, volume, assumes shape of containing vessel.

   d. Gases: definite mass, but neither definite volume nor definite shape.

   e. Classification of matter as solid, liquid, or gas usually made on the basis of the state at room temperature, 20 °C, or 68 °F; this implies a dependence of state on temperature.

   f. Gas vs vapor; vapor is the name given the gaseous state of a substance which exists as a liquid or solid at room temperature.

   g. Change of state is a physical change.

   h. Mass is conserved in a change of state. EXPERIMENT*

5. A MODEL of matter based on physical properties.

   a. What a model is in science --- the scientist as a model builder.

   b. Indirect investigation: BLACK BOX MODEL. EXPERIMENT*

   c. Propose a Particle Model.

   d. Evidence of particulate nature of matter from crystal formation. EXPERIMENT*

   e. Particle Model and change of state. EXPERIMENT*

   f. Particle Model and other physical properties.

   g. The MOLECULE: name of the small particle that determines the physical properties of matter.

   h. Molecular size. EXPERIMENT*

   i. The molecule and compounds.

C. Chemical properties

1. Characteristics of a substance or substances in the presence or production of other substances.

   a. Relate to transformation of a given substance into other substances.

   b. Relate to the behavior of a substance in combining or decomposing.

Experiment: MOLECULAR SIZE DETERMINATION

Before students proceed to the indirect measurement of the size of an oleic acid molecule (PSSC Monolayer Experiment), an analogous macroscopic experiment serves to clarify the method. Peas, marbles, beads, BB's, ball bearings, or similar spherical (or nearly so) object may be used; peas have an advantage if glass rather than plastic wear is used for the measurements.

(1) Pack a single layer of spheres into a Petri dish whose diameter is measured.

(2) The spheres form a "pancake" or flat cylinder whose base is given by the area of the Petri dish bottom and whose height is the diameter of single sphere, the dimension sought here indirectly.

(3) Transfer the spheres into a graduate cylinder of appropriate size to measure the total volume occupied by the spheres; this also corresponds to the volume of the "pancake" or flat cylinder.

(4) Since now the volume and the base area of the flat cylinder are known, the cylinder height can be calculated; in this case it gives the diameter of the spheres, the object of this exercise.

(5) The accuracy of the method can be readily tested by a more direct measurement of the sphere diameter; many plastic rulers provide a convenient groove into which a number of spheres can be fitted nicely along a line.

(6) The only difference between the above procedure with "macrospheres" and the measurement of an oleic acid molecule is in the determination of the total volume of the flat cylinder (thin film of oleic acid on water); here the volume is predetermined by the known dilution of the oleic acid and a measurement of the average drop size.

C. Chemical properties

2. Chemical change — EXPERIMENT*

   a. Change in which the identity of the original substance or substances is lost.

   b. New substances with new properties are formed.

   c. EXAMPLES: nails rust
      fuels burn
      cider ferments
      milk sours
      silver bromide reduces to silver

   d. Light, heat, and electricity provide the usual means to bring about chemical changes.

   e. In a chemical change, mass is conserved. — EXPERIMENT*

3. Chemical change as evidence for ATOMS,

   a. Molecules therefore not the ultimate particles of matter, although they are ultimate for a given substance.

   b. Changing from one substance to another requires a change in the molecules of the original substance.

   c. An atom is to an element what the molecule is to a compound.

   d. Read: CHEMS, Chapter 14, pages 233-234, "Why we believe in atoms".

4. Atomic and molecular mass: the MOLE

   a. Suppose an instrument made it possible to directly measure the mass of an atom, or of a number of atoms (while its operation is quite different from a laboratory balance, the MASS SPECTROGRAPH does provide a reliable means of measuring the masses of atoms and molecules).

   b. If a number (say, 100) hydrogen atoms were compared to the same number of carbon atoms, the mass of the carbon atoms would be about 12 times the mass of the hydrogen atoms.

   c. If the same number of oxygen and hydrogen atoms were compared, the mass of the oxygen atoms would be about 16 times the mass of the hydrogen atoms.

   d. Continuing to other elements, always comparing to the mass of the same number of hydrogen atoms:

      aluminum atoms about 27 times the hydrogen atoms' mass;
      iron atoms about 56 times the hydrogen atoms' mass;
      silver atoms about 108 times the hydrogen atoms' mass;
      gold atoms about 197 times the hydrogen atoms' mass;
      lead atoms about 207 times the hydrogen atoms' mass;
      uranium atoms about 238 times the hydrogen atoms' mass.

CHEMICAL CHANGE, COMPOUNDS, AND CONSERVATION OF MASS

Several reactions suggest themselves as particularly valuable in advancing the concept of chemical change in compound formation or decomposition. In selecting demonstrations and student experiments for this purpose, several factors should be considered:

(1) Are the reactants chemically simple?

(2) Can the reaction equation be simply and directly interpreted?

(3) Do the properties of the reactants admit of uncomplicated comparison before and after the chemical change?

(4) Are the changes in properties very obvious ones?

(5) Is it possible to make meaningful mass measurements before and after the reaction?

(6) Is the reaction safe for student experimentation?

It can hardly be expected that any single example will ideally satisfy all these criteria but several demonstrations and experiments collectively, each possibly underscoring one particular aspect, can establish a firm experimental foundation for the desired concepts. With increased attention paid to quantitative detail, some traditional exercises serve this purpose well. The following are suggested for consideration:

(1) electrolysis of water.

(2) decomposition of mercury oxide.

(3) formation of iron sulfide.

(4) formation of copper sulfide.

(5) formation of iron oxide.

(6) reaction of lead nitrate with sodium iodide.

(7) reaction of zinc with hydrochloric acid.

(8) formation of carbon dioxide by selzer tablet in water.

(9) reaction of carbon dioxide in lime water.

(10) formation of magnesium oxide.

C. Chemical properties

4. Atomic and molecular mass: the MOLE (continued)

e. Since the same number of atoms were compared in each case, these results establish a scale of RELATIVE masses of the atoms, relative because it arbitrarily assigns a value of one to hydrogen and the others are compared on the basis of this choice.

f. There are reasons (which may or may not warrant discussion here) for a choice other than hydrogen as one for establishing a scale of relative masses of the atoms: since 1961, an ISOTOPE of carbon, C^{12}, has been designated as the basis for the comparison of masses; the mass of C^{12} is taken as 12.00000 (this has the slight effect of giving naturally occurring hydrogen a relative mass of 1.00797).

g. Now that a RELATIVE scale for the masses of atoms is squared away, how about linking this idea to grams, or some other known and useful unit of mass?

h. Since all of these relative mass values have been accurately measured and tabulated (to at least five places), one clever thing that could be done is this:

(1) assemble enough atoms of C^{12} to give a mass of exactly 12.00000 grams;

(2) then that SAME NUMBER of hydrogen atoms will have a mass of 1.00797 grams;

(3) that SAME NUMBER of oxygen atoms will have a mass of 15.9994 grams;

(4) and that SAME NUMBER of gold atoms will have a mass of 196.967 grams, etc.

i. The particular number of atoms of each element that is needed to give this gram-scale of atomic masses can be found in various ways; its value is 6.02×10^{23}, and it is known as AVOGADRO'S NUMBER.

j. Recall that elements (atoms) combine to form compounds (molecules), or compounds (molecules) decompose to form elements (atoms); because mass is conserved in these chemical changes, the designation of masses on the gram-scale can be extended to molecules as well as atoms.

EXAMPLES: hydrogen gas consists of diatomic molecules, H_2; each molecule is composed of two hydrogen atoms chemically combined; Avogadro's Number of hydrogen molecules will have a mass of 2 X 1.00797 grams, or 2.01594 grams.

water consists of triatomic molecules, H_2O; each molecule is composed of two hydrogen atoms and one oxygen atom chemically combined; Avogadro's Number of water molecules will have a mass of 2 X 1.00797 grams + 15.9994 grams, or 18.0143 grams.

DEMONSTRATION*

THE MOLE

Firmly establishing the MOLE concept is the principal goal of the entire unit on matter. It is believed that the equations of chemical reactions take on more meaning and much more readily admit of quantitative interpretation if they are introduced in a "molar atmosphere".

To lend reality to the MOLE idea, a MOLE COLLECTION can be made and displayed:

> 18 grams of water (in a transparent plastic vial) is a "watermole"; appropriately sized cubes of available metals like copper, aluminum, iron, magnesium, and lead become "Cumoles", Almoles", "Femoles", etc.; mole-sized rock salt crystals can be cleaved from larger chunks. Students find the "mercury mole" incredible and bring down upon themselves an avalanche of problems predicting and comparing molesizes using nothing more than density and atomic mass.

CHEMS Experiment 7 is an excellent experimental springboard for discussing the mole in equations of chemical reactions. The prohibitive investment in silver nitrate can be avoided if lead nitrate is used instead, with zinc strip replacing the copper wire. Of course, then the theoretical Zn/Pb ratio of moles is 1.00 rather than 2.00.

Experiments already performed provide a wealth of examples for further discussion and problems: formation of iron sulfide, burning of magnesium ribbon, electrolysis of water, etc.

C. Chemical properties

4. Atomic and molecular mass: the MOLE (continued)

k. Avogadro's Number of particles, whether they are atoms or molecules, is also known as a MOLE.

EXAMPLES: one mole of atomic hydrogen contains 6.02×10^{23} atoms and has an atomic mass of 1.00797 grams;

one mole of hydrogen gas contains 6.02×10^{23} molecules and has a molecular mass of 2.01594 grams;

one mole of gold contains 6.02×10^{23} atoms and has an atomic mass of 196.967 grams;

one mole of water contains 6.02×10^{23} molecules and has a molecular mass of 18.0143 grams;

one mole of aluminum contains 6.02×10^{23} atoms and has an atomic mass of 26.9815 grams; DEMONSTRATION*

one mole of sodium chloride contains 6.02×10^{23} molecules and has a molecular mass of 58.442 grams. DEMONSTRATION*

l. The MOLE in chemical reactions EXPERIMENT*

D. Density: an important property of matter

1. Mass density

a. Defined as MASS PER UNIT VOLUME, or $D = m/V$, where m is the sample mass and V is its volume. EXPERIMENTS*

b. Common units of mass density are grams/cm^3; referring to liquids, it is often expressed equivalently in grams/milliliter; for gases, grams/liter is in common use.

c. Note that the units of mass density in the MKS system are kg/m^3.

d. Because of the original definition of mass, the mass density of pure water is 1.0000 grams/cm^3 (at a specified temperature).

e. Numerical value of the mass density is given by the mass (in grams) of one cubic centimeter of a substance. DEMONSTRATION*

f. Mass density is commonly referred to simply as DENSITY, but it should be distinguished from weight density.

Purpose: to find the thickness of (thin) aluminum foil using a small sample mass measurement and the given density of pure aluminum.

(A suggested practice is to give to the students only the purpose of the experiment, none of the following procedure. A carefully planned outline of procedure is required, however, before the student begins the laboratory work.)

(1) Cut several different "squares" of the same aluminum foil. To be of proper size for the millibalance, these should be about 1 - 3 cms on a side.

(2) Obtain the mass of each "square" in margs on the millibalance; calculate the mass of each in grams.

(3) Use the HANDBOOK OF CHEMISTRY AND PHYSICS or other appropriate source to find the density of pure aluminum (2.69 - 2.70 grams/cm^3)

(4) Since a "square" of foil has a volume given by the side s squared times the thickness t, and density D is defined as the mass divided by the volume, the thickness is obtained by dividing the measured mass m by the product of the side squared times the density or

$$t = m/s^2D.$$

(5) A value for the thickness is to be obtained for each of the squares and the results compared and averaged.

(6) A comparison can also be made between regular and heavy duty foil.

QUANTITATIVE SCIENCE NAME________________________

APPROXIMATE MASS DENSITIES OF SEVERAL COMMON SUBSTANCES (at ordinary temperatures)

SUBSTANCE	MASS DENSITY (grams/cm^3)
gold (Au)	19.3
mercury (Hg)	13.6
lead (Pb)	11.3
steel	7.5 - 7.8
aluminum (Al)	2.7
glass	2.4 - 2.8
rock salt (NaCl)	2.18
carbon tetrachloride (CCl_4)	1.595
sugar	1.59
carbon disulfide (CS_2)	1.29
polystyrene	1.04 - 1.06
milk	1.028 - 1.035
water	1.000
ice	0.917
benzene	0.899
cherry wood	0.70 - 0.90
cedar wood	0.49 - 0.57
cork	0.22 - 0.26
styrofoam	0.025 - 0.080
air (earth surface)	0.0012
air at 20 km	0.00009

QUANTITATIVE SCIENCE

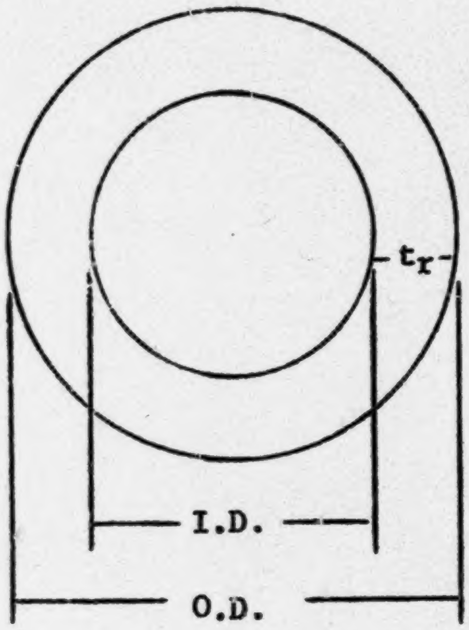
INSTRUCTOR INFORMATION SHEET

EXPERIMENT: Density of Wax Paper (or Plastic Food Wrap)

Purpose: to find the density of wax paper (or plastic wrap) using a small sample mass measurement and an estimated thickness.

(It is suggested that only the purpose of the experiment be given to the students. It may, however, be necessary to give limited guidance to some students on obtaining a good value for the estimated thickness)

1. A very good value for the thickness can be obtained rather easily by using a roll of wax paper as it is purchased: a full, tight, roll of stated length.



2. Although the number of turns per roll can be counted, it is quite cumbersome (the count is usually well over 200). To avoid this, it is only necessary to measure the inside and outside diameters, I.D. and O.D., respectively, of the wax paper roll. From these, an average diameter d_{av} of each turn of wax paper can be calculated:

   $$d_{av} = (\text{I.D.} + \text{O.D.})/2,$$

   and an average length for each turn is obtained by the product πd_{av}.

3. The known roll length L is given by the total number of turns n times the average length of one turn, or $L = n(\pi d_{av})$. Hence, the number of turns n can now be found.

4. The roll thickness t_r (given by the difference of the O.D. and I.D. divided by 2) divided by n gives the thickness t of a single sheet of wax paper:

   $$t = t_r/n.$$

5. Cut convenient "squares" of wax paper (from another but identical roll); measure the side length s.

6. Measure the mass of each "square" on the millibalance; express the mass m in grams.

7. Since the density D is given by the mass divided by the volume, D is obtained here by dividing the measured mass m by the volume s^2t, or

   $$D = m/s^2t.$$

8. The density is to be found for each "square" and the results compared and averaged.

D. Density: an important property of matter (continued)

2. Weight density
   - a. Defined as WEIGHT PER UNIT VOLUME.
   - b. Weight density is commonly used only in the English system.
   - c. Units of weight density are lbs/ft^3; what would you expect the units to be in the metric system?
   - d. Numerical value of the weight density is given by the weight (in lbs) of one cubic foot of the substance.
   - e. Weight density of pure water is 62.4 lbs/ft^3.

3. Specific gravity
   - a. Ratio of the density (mass or weight density, but be consistent) of a given substance to the density of water: $S.G. = D_{substance} / D_{water}$
   - b. Also, specific gravity is the ratio of the mass (or weight) of a given substance to the mass (or weight) of an equal volume of water. DEMONSTRATION*
   - c. Specific gravity is a PURE NUMBER; it is a dimensionless quantity; it has no units (WHY?)
   - d. Specific gravity of any substance is NUMERICALLY equal to the value of the MASS DENSITY of that substance expressed in grams/cm^3
   - e. EXAMPLES:

     specific gravity of purewater is 1.00.
     specific gravity of aluminum is 2.70.
     specific gravity of lead is 11.3.

4. What has been discussed to this point might better be referred to as SAMPLE density.
   - a. If the sample is not homogeneous or is not the same throughout, dividing the sample mass by the volume will give the AVERAGE density of the sample.
   - b. Contrast this with LOCAL density. EXPERIMENT*

     EXAMPLES: raisins in a pudding have a different density than the pudding.

     earth's crust has a different density than the earth.

Experiment: DENSITY OF IRREGULAR SOLIDS BY THE SLOPE OF A LINE

Graphing cumulative mass vs cumulative volume can be used to determine the density of a variety of irregular materials such as:

aluminum shot,
copper turnings,
marble chips,
solder scraps,
plexiglas pieces, etc.

The laboratory balances are used to measure the mass of one (or several, depending on size) piece of the particular material being investigated; the corresponding volume is determined by the displacement of water in a graduate cylinder. Consecutively adding the mass of each piece to the previous total gives the cumulative mass which is plotted against cumulative volume, read directly if the pieces are merely left in the graduate cylinder. The slope of the resulting line gives the density of the material.

The procedure is even simpler for liquids, powders, grains, etc.

QUANTITATIVE SCIENCE NAME________________________

1. a. Convert g/cm^3 to kg/m^3.
   b. Express the density of water in the MKS system.
   c. Express the density of lead in the MKS system.

2. What is the mass of a bar of pure gold measuring 20 cm by 12 cm by 4 cm?

3. How thick is a piece of plate glass that has a mass of 3.42 kg and measures 0.421 meters X 0.620 meters?

4. What is the density of a sphere that has a mass of 9.2 kg and a radius of 9.2 cm?

5. An AMALGAM of gold and mercury is made by alloying 3.2 kg of gold and 1.4 kg of mercury. Assuming that there is a 10% decrease in the total volume after the amalgamation, what is the density of the amalgam?

6. a. What is the weight density of rock salt?
   b. Find the weight of 15 ft^3 of styrofoam.
   c. A copper rod 16 feet long and 6" in diameter weighs 1755 pounds. What is the specific gravity of copper?

7. A gallon of a certain liquid weights 9.62 pounds. What is its density in g/cm^3?

8. Use the HANDBOOK OF CHEMISTRY AND PHYSICS (or other references) to find the densities of the following substances:

   a. beryllium
   b. bromine
   c. quartz
   d. sealing wax
   e. granite
   f. dry leather
   g. castor oil
   h. sea water

9. Assuming the film of a soap bubble to be almost all water, if the bubble has a mass of 0.0012 grams and a film thickness of 1.0 X 10^{-6} cm. what is the radius of the bubble?

10. Plot the following data on the density of water as a function of temperature: COMMENT

TEMPERATURE (°C)	Mass density (g/cm^3)
-1.0	0.99979
0.0	0.99987
1.0	0.99993
2.0	0.99997
3.0	0.99999
4.0	1.00000
5.0	0.99999
6.0	0.99997
7.0	0.99993
8.0	0.99988
9.0	0.99981
10.0	0.99973

QUANTITATIVE SCIENCE NAME____________________

1. A 50 ml graduate cylinder is filled to the 20.00 ml mark with water. Into it are dropped 10.00 grams of quartz (D = 2.65 g/cm^3) and 11.04 grams of beryllium (D = 1.84 g/cm^3).

   a. What is the total mass held by the graduate cylinder?

1a____________________

   b. What is the reading of the water level after the substances have been added?

1b____________________

2. A 200-pound block of ice is melted. What is the volume of the water produced if the density of ice is 0.917 grams/cm^3?

2____________________

3. A bar of pure gold (D = 19.3g/cm^3)had a mass of 12.367 grams before and 11.984 grams after it had been used to electroplate an article that had a surface area of 1600 square centimeters.

   a. How thick is the gold plate on the article?

3a____________________

   b. If the original mass of the article was 68.0 grams, what is the percentage increase in its mass after the deposition of the gold?

3b____________________

E. Solubility: another property of matter

1. Operational definition of solubility: the degree to which a pure substance when mixed with a second pure substance "enters into" or dissolves into the second substance (at a specified temperature).
   a. The product or result of this mixture is called a solution.
   b. Solute: substance that is dissolved; may be solid, liquid, or gas.
   c. Solvent: substance that does the dissolving; may be solid, liquid or gas.
   d. Solubility is a quantitative term which states three things: amount of solute, amount of solvent, and temperature.

2. LIQUIDS are most often used as solvents
   a. Water is most nearly "universal solvent"; AQUEOUS SOLUTIONS
   b. Other solvents commonly used: alcohol, acetone, benzene, carbon tetrachloride, ether, etc.

3. True solution --- even though it is a MIXTURE --- does not separate into its component parts on standing
   a. Solute does not PRECIPITATE.
   b. Solutions are NOT suspensions, emulsions, colloids.
   c. Solution contains the solute uniformly distributed throughout the solvent.
   d. True solutions are generally clear but may have some color.

4. Vast majority of chemical changes in nature take place when substances are in solution
   a. DIGESTION of food as an example.
   b. Many substances when dry do not react at all (or do so very slowly), but in solution react very quickly, sometimes even violently.

5. Range of solubility; precise definition of solubility
   a. Range extends from INSOLUBLE to EXTREMELY SOLUBLE.
   b. There is a precise limit to the amount of a substance that will dissolve in a given amount of a solvent at a particular temperature.
   c. Common, precise way of expressing SOLUBILITY: number of grams of solute that will dissolve in 100 grams of solvent (usually water) at a specified temperature.

   EXAMPLES: 37 grams of NaCl will dissolve in 100 grams of water at 25°C; 0.0014 grams of $CaCO_3$ will dissolve in 100 grams of water at 25°C.

6. Factors affecting solubility and the RATE of solubility
   a. Temperature; generally, increased temperatures increase solubility. SOLUBILITY CURVES (see HANDBOOK OF CHEMISTRY AND PHYSICS)
   b. Agitation (stirring, shaking, etc); effective since agitation removes the newly formed solution from the solute and thereby re-exposes the solute to the solvent.
   c. Pulverization (of solids): powdered substance dissolves faster than same material in bulk form.

QUANTITATIVE SCIENCE DATE________________ NAME_________________________

1. Find the solubility of the following substances as listed in the HANDBOOK OF CHEMISTRY AND PHYSICS.

   a. silver nitrate at 20°C.
   b. barium nitrate at 40°C.
   c. cyclohexanol at 15°C.
   d. 1,2 - dichlorobenzene at 25°C.
   e. ammonium perchlorate at 0°C.
   f. calcium hydroxide at 0°C.

2. Plot the solubilities (from 0 to 100°C) of the following salts to obtain their solubility curves (grams/100 g H_2O vs temperature)

   a. KCl (potassium chloride)
   b. NH_4Br (ammonium bromide)
   c. $KClO_3$ (potassium perchlorate)

   } three curves on one graph

3. If 14.3 grams of a salt dissolve in 46.0 grams of water, how would its solubility be reported in the HANDBOOK OF CHEMISTRY AND PHYSICS?

4. What is the smallest volume of water at 20°C required to dissolve 100 grams of cobalt sulfate?

5. What percent of solid sodium chloride is contained in a saturated sodium chloride solution at 0°C.?